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ABSTRACT

Part One of this report discusses the background and philosophy of the Patterns in Arithmetic (PIA) television program. Mathematical objectives for Grades 1-4 are discussed in detail. Part Two presents the results of a summative, noncomparative evaluation done on the PIA program in Grades 1 and 3 during the 1966-67 school year. Classes in Alabama and Wisconsin participated in the study at both grade levels. The instruments used in the evaluation fall into three categories: (1) achievement tests designed specifically for the PIA program; (2) standardized achievement test; and (3) teacher and pupil inventories. (MP)

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Technical Report No. 67

Objectives of Patterns in Arithmetic and Evaluation of the Telecourse for Grades 1 and 3

Report from Phase 1 of the Project on Prototypic Instructional Systems in Elementary Mathematics

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THE UNIVERSITY OF WISCONSIN
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Technical Report No. 67

OBJECTIVES OF *PATTERNS IN ARITHMETIC* AND EVALUATION OF THE TELECOURSE FOR GRADES 1 AND 3

By James S. Braswell and Thomas A. Romberg

Report from the Project on Individually Guided Instruction in Elementary Mathematics
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STATEMENT OF FOCUS

The Wisconsin Research and Development Center for Cognitive Learning focuses on contributing to a better understanding of cognitive learning by children and youth and to the improvement of related educational practices. The strategy for research and development is comprehensive. It includes basic research to generate new knowledge about the conditions and processes of learning and about the processes of instruction, and the subsequent development of research-based instructional materials, many of which are designed for use by teachers and others for use by students. These materials are tested and refined in school settings. Throughout these operations behavioral scientists, curriculum experts, academic scholars, and school people interact, insuring that the results of Center activities are based soundly on knowledge of subject matter and cognitive learning and that they are applied to the improvement of educational practice.

This Technical Report is from Phase 1 of the Project on Individually Guided Instruction in Elementary Mathematics in Program 2. General objectives of the Program are to establish rationale and strategy for developing instructional systems, to identify sequences of concepts and cognitive skills, to develop assessment procedures for those concepts and skills, to identify or develop instructional materials associated with the concepts and cognitive skills, and to generate new knowledge about instructional procedures. Contributing to the Program objectives, the Mathematics Project, Phase 1, is developing and testing a televised course in arithmetic for Grades 1-6 which provides not only a complete program of instruction for the pupils but also inservice training for teachers.

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ABSTRACT

This report is divided into two main parts. Part One discusses the background and philosophy of the Patterns in Arithmetic (PIA) TV program. Mathematical objectives for Grades 1–4 are discussed in detail.

Part Two of the report presents the results of a summative, noncomparative evaluation done on the PIA TV program in Grades 1 and 3 during the 1966–67 school year. Classes in Alabama and Wisconsin participated in the study at both grade levels. The instruments used in the evaluation fall into three categories: (1) achievement tests designed specifically for the PIA program, (2) standardized achievement tests, and (3) teacher and pupil inventories.

In addition to the state variable the PIA staff was interested in how operational the program was in various socioeconomic groups and community sizes. Each class was classified as belonging to one of four community sizes. If a class belonged to one of the two larger community size categories, it was also classified into one of three socioeconomic categories: high, middle or low.

In the first grade the study group compared quite favorably with the norms group on both standardized computation and concepts tests. This was true for both Alabama and Wisconsin classes considered separately as well as combined. In addition, tests were constructed by Educational Testing Service for PIA to measure program achievement. These tests were generally difficult, but the results show that most of the objectives set forth by the PIA staff are being accomplished.

The statistical analysis of data was done using multivariate analysis of variance. The hypothesis that there were no significant differences across the four community size groups was not rejected. The hypothesis that there was no significant difference between states was rejected. The difference between states favored the Wisconsin group on the standardized computation test, but not the standardized concepts test. The hypothesis that there were no significant differences across socioeconomic groups was not rejected. Results tended to favor the middle and high socioeconomic groups over the low socioeconomic group. The differences were more pronounced on the standardized computation and concepts test than on the tests designed specifically for PIA.

In the third grade the PIA group also compared favorably with the norms group. Pretesting done in Grade 3 showed both Alabama and Wisconsin performing slightly better than the norms group on the standardized concepts test, but comparing unfavorably with the norms group on the standardized computation test. During the year reasonable growth was made by children in both Alabama and Wisconsin and across the various socioeconomic groups. By the end of Grade 3 the Alabama children were clearly better than the norms group on both computation and concepts tests. The Wisconsin children did clearly better than the norms group on the concepts test, but were slightly below norms on the computation test. This latter result is due to the fact that at the beginning of Grade 3 the Wisconsin group was well below norms on computation. They did show considerable improvement over the year. As for the first grade, the tests designed specifically to measure program achievement were generally difficult. However, item analysis showed the major objectives of PIA being accomplished.

The statistical analysis of data in Grade 3 used a pretest as a covariate. For some classes the pretest was a concepts test, while for others it was a computation test. In either case the hypothesis that there were no significant differences across the four community size groups was not rejected. There was a highly significant difference between states in favor of Wisconsin when the computation test was used as a covariate. The Wisconsin group did significantly better on tests developed by Educational Testing Service to measure program achievement. When a computation test was used as a covariate there was no significant difference across the three socioeconomic groups. However, when the concepts test was used as a covariate, there was significant difference between socioeconomic groups. It appears that even with scores adjusted by covariance, the PIA program operated better within the high socioeconomic group than the low or middle.

In both grades the opinion inventories showed that teachers and children liked TV arithmetic. Teachers felt that the concepts covered were appropriate and reasonably paced. The children felt the TV teacher helped them learn arithmetic and they enjoyed working on the worksheets. The first-grade program appeared to be aimed at middle ability children, while the third-grade program was for middle to high ability children. One major objective of the TV tapes is to provide inservice training for teachers as well as instruction for children. Response to the Teacher Opinion Inventory indicates PIA is an effective inservice instrument.

PART ONE
OBJECTIVES

I
BACKGROUND

THE HISTORY OF PATTERNS IN ARITHMETIC

In 1959 the Ford Foundation gave the University of Wisconsin a substantial grant to establish an imaginative program for the improvement of the schools of Wisconsin. Under the able leadership of Dr. John Guy Fowlkes, the organization developed to carry out the terms of the grant became known as the Wisconsin Improvement Program (WIP). The first sequential television course in mathematics was developed with generous financial support from WIP. In 1960, Patterns in Arithmetic, a complete program of instruction for Grade 4, was developed and Patterns in Mathematics, a sequential course of instruction for first year algebra, was developed also.

In 1961 a grant from the National Science Foundation made it possible to continue to develop a sequence of television lessons for Grades 5 and 6. During the period 1961–1963 the television lessons, with accompanying manuals and exercise books, were developed for these grades. Much of the success of this program was due to the excellent cooperation received from the schools in the WHA-TV, Channel 21, and the WMVS, Channel 10, areas and, in particular, from the administrators and teachers in Milwaukee and Madison.

When the Wisconsin Research and Development Center for Cognitive Learning (R & D Center) was established at the University of Wisconsin, it seemed desirable to develop a six-year program in mathematics based on past experience with the projects mentioned above. It now became possible to extend the program to Grades 1, 2, and 3 as well as to improve the programs previously taped for Grades 4, 5, and 6. Furthermore, through the R & D Center's facilities, it now became feasible to establish a more systematic field testing of the PIA program than had been possible in previous attempts to develop a sequential television program. The extension of the efforts to develop a television program in arithmetic based on sound educational practices will be set forth in the following sections of this report.

WHY TEACH ARITHMETIC BY TELEVISION?

The revolution in mathematics and the rapid development of the industrial and scientific uses of mathematics following World War II caught the elementary schools, and in fact the education system from Grade 1 to 16, by surprise. In spirit and in body, the mathematics courses were not those that were needed for the further industrial and scientific growth of the nation. The spirit of the old elementary mathematics was too heavily loaded with computational devices with too little emphasis on the fundamental ideas of mathematics. As a result, the courses were drill oriented and children were not given the opportunity to see that mathematics could be enjoyable. The "revolution" made it imperative that new ideas be introduced into the elementary school classroom.

The administrator faced with the problem of introducing new ideas into the classroom has two principal avenues available. He can (1) encourage the teachers to get further training at some university or college or (2) provide inservice training. The merits of these approaches to staff improvement will be considered separately.

It is, of course, desirable for teachers to continue their study of the teaching process and the subject that they are teaching. However, it is not always possible for a teacher to go to summer school, and it is not always possible for the administration to require that teachers return to school for further training. Furthermore, the additional training that teachers get in universities and colleges is not usually focused on the particular problems of a teacher or a school. These conditions give

rise to a certain amount of wasted time and money.

Inservice training programs can suffer from some of the same defects found in further study at a university. However, inservice courses can be focused on the particular problems of a school more readily than credits earned at an institution of higher learning. The administration must keep in mind that inservice courses place a heavy load on the teaching staff since these courses must be given after school or in the evenings. This places an added burden on an already heavy teaching load. While inservice training can be effective, it is far from ideal or even reasonable if carried on at the expense of increasing the teacher's work load.

The upgrading of a teaching staff is always a difficult task. This task is made even more difficult by the usually high turnover of staff in the elementary school. It becomes very difficult to improve large staffs with an annual turnover of 25% to 50%.

For large systems, communicating with masses of teachers is a real problem. This problem is amplified manyfold when one wants to communicate ways of teaching a given topic or to communicate an entirely new organization of a given discipline. Even with the best of supervisory staffs it is physically impossible to spend even one-half hour a month visiting with each teacher. Even meeting the teachers in each building on any systematic basis places a heavy strain on the time of a supervisory staff. In view of the realities of the school situation it would seem that some other method must be devised to communicate with large masses of teachers about the newer ideas in mathematics for the elementary school and to demonstrate a change in the spirit of teaching the subject.

If the elementary schools of the nation are to make faster progress in improving their mathematics programs, it seems quite reasonable to assume that modern methods of mass communication might well be used. It has been found that teaching arithmetic by television can be instrumental in changing school programs in two ways. (1) It provides an inservice course for the teacher without requiring her to attend evening classes, summer schools, or after school meetings. (2) It provides a sequential course in arithmetic for the pupil. Each of these functions of Patterns in Arithmetic will be more completely described in the following sections of this report.

II

A DESCRIPTION OF PIA

The PIA program is planned to maximize classroom teacher and pupil participation. The classroom teacher is an important member of the TV teacher-classroom teacher team. The team idea is built into the program by providing two television lessons per week with accompanying pupil exercises and teacher notes. The television teacher teaches the class for one-half hour per week. The regular classroom teacher teaches the class for the remaining time. Through an arrangement of this kind, the classroom teacher is an active participant in the total instructional program. It is this feature of PIA that provides the in-service program for the teacher. Teachers learn how to teach the new arithmetic by participating in the teaching act. This is the best way to learn how to teach.

Each television program in the series is accompanied by lesson notes for the teacher and exercises for the pupil. The exercise books for the pupil provide enough work for almost all classes. The teacher may need to select carefully for the weaker student and provide some enrichment material for the academically strong pupil.

The "way to teach" function of PIA necessitates that the program give complete coverage of all of the major ideas of arithmetic found in the schools plus some of the new content that is currently finding its way into the curriculum. Consequently, PIA is not suitable for use as an enrichment program only. If PIA is to be used, it should be used in its entirety and used alone. The concepts are sequenced so as to make learning easier. Since it integrates with no other pupil book on the market, it cannot be readily used in conjunction with another program. *PIA was not constructed as an enrichment program.* It is the entire program in arithmetic. If it is to be supplemented in any way, the supplementary materials should conform to the sequence of concepts found in PIA.

The fact that PIA is a complete arithmetic program and not supplementary can be gleaned from the following brief description of the program for each grade.

BASIC MATHEMATICAL IDEAS, GRADE 1

The basic mathematical ideas presented in PIA 1 are the foundation upon which the ideas of the later grades are built. One-to-one correspondence sets the stage for the ideas of cardinal number and the relations "as many as," "more than," and "fewer than." The operation of addition of whole numbers is based on the set operation of union, through which the commutative property of addition is presented. This property is used to facilitate the learning of the addition facts which are carried through sums to 10. Subtraction of whole numbers is based on the complement of sets and is carried through minuends of 10. Numeration and the natural order of the whole numbers are carried out through 99. The monetary system is introduced as an application of the numeration system, and the relationships between penny, nickel, dime, and quarter are stressed. Linear measurement is given attention by considering arbitrary units of measure with the standard units as a special case. The geometrical concepts of point, curve, open curve, closed curve, betweenness, segment, and common closed curves (triangles, rectangles) are all considered. The ordinal numbers through "tenth" are fused with the respective cardinal numbers. Mathematical sentences and solution sets are introduced along with addition and subtraction.

BASIC MATHEMATICAL IDEAS, GRADE 2

Many of the concepts presented in Grade 2 are extensions of those already met in Grade 1. Numeration and order of cardinal numbers are extended through 999. The operations of addition and subtraction are also extended. Sums

to 999 and associated differences are introduced, using the appropriate algorithm. Addition facts through 18 are to be mastered as well as the associated subtraction facts. The commutative and associative properties of addition and the additive property of zero are extended or introduced as the case may be. The operation of multiplication is introduced using facts with products through 18. The commutative property of multiplication is used to facilitate learning of these facts. The geometrical concepts of point, line, polygon, closed curve, open curve, and region are extended, and the concept of a geometrical transformation is introduced. Linear measurement is extended with special emphasis on the inch and the foot. Perimeters of polygons and their measures are also introduced. Mathematical sentences and solution sets are extended with emphasis on both equalities and inequalities. These sentences are presented in conjunction with solving problems. The monetary system and ordinal numbers are also extended.

BASIC MATHEMATICAL IDEAS, GRADE 3

Many of the concepts presented in Grade 3 are extensions of those already presented in Grades 1 and 2. Numeration and order of cardinal numbers are extended through 9999. Base 5 and Base 8 numeration systems are introduced as enrichment work. Addition and subtraction are extended. Sums of two or three numbers greater than 10 but less than 10000 are introduced. Differences for numbers less than 10000 are also introduced. Study of the commutative, associative and zero properties for addition is continued. Multiplication is extended with a review of the facts with products through 18. The concept of a factor is introduced. The commutative, associative, and distributive properties of multiplication and the multiplicative property of 1 are extended or introduced, as the case may be. These are used to find products such as 3 x 24. Division is defined in terms of multiplication. Mathematical sentences and solution sets are used extensively in problems involving multiplication, addition, subtraction, and division. Geometrical concepts are extended with particular emphasis being given to line segments, closed curves, angles, triangles, the inside and the outside of curves, polygons, similarity and congruence for polygons, congruence for angles and segments, and an introduction to the circle. The function concept is introduced under the guise of number patterns. Ratios and fractions are introduced, linear measure is extended, and liquid measure is introduced. Even and odd numbers are related to factors.

BASIC MATHEMATICAL IDEAS, GRADE 4

Many of the concepts presented in Grade 4 are extensions of the concepts of the lower three grades. Numeration and order of cardinal numbers are extended through billions. Addition and subtraction and appropriate properties are reviewed. Multiplication is extended to such products as 5 x 38, 23 x 47, and 23 x 382. All multiplication facts through 9 x 9 are to be mastered. Here the commutative property is utilized. The associative and distributive properties are utilized when finding products like 3 x 24. Division is extended and related to multiplication. The division algorithm as repeated subtraction is extended to three-digit divisors. Division exercise involving remainders are included. Mathematical sentences are used to solve problems involving multiplication, division, addition, subtraction, and ratios. Equivalent sentences are introduced. Sets of ratios (linear vector space) are discussed prior to fractions, sets of fractions, and basic fractions. Sets of fractions are associated with the number line. Factors and prime and composite numbers are extended or introduced as the case may be. Linear and liquid measures are extended. Estimation of products and sums as well as approximate measures are introduced. Geometrical concepts are extended to include parallel and perpendicular lines, vertices, edges and faces of polyhedra, congruencies and similarities, perimeters, rotation, slides and flips, and classification of quadrilaterals.

THE KEY MATHEMATICAL IDEAS OF PIA

Any modern mathematics program for the elementary schools is based on certain key mathematical ideas that permeate the entire course of instruction. In PIA these ideas are spirally arranged throughout the course of instruction. The ideas on which PIA is based are briefly described in the following paragraphs.

1. Set

The concept of a set is fundamentally for developing and communicating ideas in mathematics. As early as Grade 1 pupils become

familiar with the simple concepts which involve sets.

2. Number

One of the main features of the program is the logical development of natural numbers (counting numbers) and the rational numbers.

3. Numeration System

After counting with some degree of understanding has been achieved with both cardinal and ordinal numbers, the pupils are taught how to write the numbers 0-9 and the concept of place value is introduced.

4. Operation

A considerable portion of elementary school arithmetic is concerned with the four fundamental operations: addition, subtraction, multiplication, and division. Of course, one of the major objectives of any elementary school arithmetic program is to develop accuracy and speed in computing. However, computing in itself is not enough. Hence PIA has placed considerable emphasis on the ideas upon which computing is based in an attempt to make computation more than rote calculation. For example, in forming the sum 3 + 2 the pupil considers a stationary set of 3 objects and another set of 2 objects which appear to be joining the given 3. Thus 3 + 2 is looked upon as a set of 3 being joined by a set of 2 to form a set of 5. The concept of column addition is approached through tally charts in an attempt to impart some understanding of place value. Rectangular arrays are used to analyze geometrically the operation of multiplication.

5. Mathematical Sentence (Equations and Inequalities)

Since language is the vehicle through which one communicates ideas, it is important that pupils begin to develop the ability to generate clear and precise mathematical sentences as soon as possible. In the first grade the pupil encounters many experiences with pictures and objects which lead to basic sentence forms. In word problems, pupils are requested to "write a sentence which tells the story of the problem" before finding the solution. The following is an example selected from the third grade: "A box holds 6 apples. How many boxes are needed for 30 apples?" The sentence associated with this problem is N x 6 = 30.

6. Measurement

Due to its importance in everyday life as a key link between our mathematical and social environment, measurement is systematically studied, beginning in the first grade of PIA. As an introduction the first grade pupil becomes familiar with the concept of relative length (the desk is less than five pencil-lengths) and nonstandard units of measure (the pencil above). Upon completing the first four grades, the pupil will be able to carry out approximate linear measurements in standard units (inches, feet), and he will be able to find the perimeter of some elementary geometrical forms (triangle, rectangle).

7. Geometry

One of the unique features of the program is a systematic development of elementary geometrical concepts beginning in the first grade. Aside from learning the names of the more common geometrical figures, the pupil becomes familiar with open and closed curves, interior and exterior of geometrical forms, points, lines and angles, intersection of curves, parallelism, perpendicularity, and some simple geometric transformations.

8. Number Theory

Special properties of numbers are developed and used to facilitate learning of arithmetic and to make the subject enjoyable. These properties include odd and even numbers, primes, composites, factors, and unique factorization.

9. Practical Aspects

Although the practical aspects should not perhaps be mentioned as a fundamental strand, these aspects are important enough in the daily activities of pupils to warrant special attention. Beginning in Grade 1 the pupil is taught linear measure and capacity. By the end of Grade 4 the pupil will have some experience with measuring the boundary and

area of plane geometrical forms. Other topics covered in the first three grades are money and making change, telling time, and use of the thermometer.

SOME PEDAGOGICAL PRINCIPLES UNDERLYING PIA

Every course of instruction is based on either an explicit or implicit set of ideas about how children learn. PIA is no exception. Through the activities of the R & D Center, and through the activities of many other researchers in the field, certain principles of learning have been established relative to arithmetic. These principles have been taken into consideration in constructing the sequence of television lessons which constitutes PIA. Some of the main elements are the following:

1. Children arrive at mathematical abstractions by observing events in the physical world. These events include not only sets of objects which are assigned a number, or some other mathematical entity, but also the physical actions that are implicit in many mathematical concepts. For example, the concept of addition is based on the idea of the union (joining) of two sets. To illustrate the idea of union certain actions are involved. Thus, if the sets involve 3 objects and 5 objects, the number of objects in the union of these sets is 3 + 5. The concept of addition is thereby associated with the union of two sets. Later the child learns to associate the standard name 8 with the sum 3 + 5, and he also learns that 3 + 5 = 5 + 3.

Furthermore, it has been established[1] at the R & D Center that children find it easier to associate the sum 3 + 5 with some physical situations involving this sum than with others. Any contemporary arithmetic program should take these facts into consideration. PIA does this by using only those physical situations (problems) that have been found the easiest during the initial stages of instruction. The harder situations are always left until the basic concept has had time to mature.

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[1]Steffe, Leslie. *The Performance of First Grade Children in Four Levels of Conservation of Numerousness and Three IQ Groups When Solving Arithmetic Addition Problem*, Research and Development Center for Cognitive Learning, University of Wisconsin, Madison, Wisconsin, Technical Report No. 14, 1966.

2. Modern theories of instruction also place an emphasis on the proper sequencing of concepts to facilitate learning. While much research needs to be done in this area, certain common sense judgments must be made without the benefit of research results. An example or two may help make this point clear.

   a. The idea of one-to-one correspondence is more fundamental than the idea of counting. Children can determine whether there are more, fewer, or just as many objects in Set A as in Set B by trying to set up a one-to-one correspondence between the objects in A and the objects of B. Later they learn how to determine the relative numerousness of two sets by counting, although in many instances in later life the idea of determining the relative numerousness of two sets by setting up a correspondence is maintained.
   b. The concept of dealing with pairs of numbers is easier to attain when dealing with such physical situations as 5 apples for 14 cents (5/14) than by cutting a pie into fourteen parts and considering five of these parts. Consequently, in PIA work with ratios (5 apples for 14 cents) is introduced before the rational numbers are introduced through cutting up pies and cakes.

There are many other instances in which basic considerations of ease of learning indicate that the traditional sequence of concepts is not ideal for learning arithmetic. The PIA staff has tried to consider all of these instances and to obtain the optimum organization of concepts for the television lessons.

3. It is generally agreed that a spiral organization of subject matter is more conducive to learning arithmetic than a nonspiraled organization. PIA does not spend more than two consecutive weeks on any one idea or skill. The skills of addition, subtraction, multiplication, and particularly division are spiraled throughout as many as three or four years. This spiraling is counter to the actual practice of many classroom teachers who like to teach all of the skills of addition of two and three digit numbers before considering other skills.

4. Motivating instruction is always a key consideration in developing a school program. PIA motivates the pupil to learn arithmetic by providing applications and as many interesting mathematical ideas as possible. An examination of the content of any grade will show that a variety of basic mathematical

ideas have been introduced. Many of these ideas can provide the basis for making arithmetic an enjoyable subject.

5. Children mature in their thought processes through a variety of contacts with ideas and through long exposure. PIA takes advantage of this psychological fact by not insisting on a "polished performance" soon after an idea or skill is introduced. The ability to compute is a good example to illustrate this principle. Carrying in addition in the hundreds is introduced early in the third grade. The teacher who insists on mastery at this time will be disappointed. The PIA program spirals this skill throughout the year, thereby avoiding long periods of drill.

These are some of the principal elements of learning that have been taken into consideration in the PIA program. The elements of instructional theory and the elements of mathematics have been interwoven so as to produce the best possible program for the improvement of the schools of the nation.

THE BEHAVIORAL OBJECTIVES

If an arithmetic program is to have a sound basis for evaluation, some statement of instructional outcomes must precede the act of teaching and the day-to-day organization of the lessons. These objectives are best stated in terms of what children are expected to do to demonstrate that they have attained the objective. The following is a statement of the content objectives and the ability objectives of the PIA program for the first four years.

Major Concepts	Student Abilities
1. Set (Set concepts will be used as a basis for the mathematics program; however, set symbolism will be minimal.)	11.1* To select a set of objects which have a given property. 11.2 To set up a one-to-one correspondence between equivalent sets. 11.3 To use one-to-one correspondence as a basis for determining "more than," "fewer than," and "same as." 11.4 See 14.1 and 14.3.
	21.1 To work with the union of overlapping sets. 21.2 See 24.3 and 24.6.
	31.1 See 39.1.
	41.1 See 42.3. 41.2 See 49.1
2. Number	12.1 To determine the cardinality of a set if $N \leq 99$. 12.2 To determine the cardinality of a set where $N \leq 4$ without counting. 12.3 To use the ordinal numbers ≤ 5. 12.4 To determine "betweenness," "less than," "greater than," and "equal to" for numbers ≤ 99. 12.5 To use informally one-half and one-fourth of a given concrete unit.
	22.1 To determine the cardinality of a set if $N \leq 999$. 22.2 To use the ordinal numbers ≤ 10. 22.3 To determine "betweenness," "less than," "greater than," and "equal to" for numbers ≤ 999. 22.4 To use informally one-half, one-third, and one-fourth of a given unit. 22.5 To use the number line to picture order of numbers.
	32.1 To determine the cardinality of a set if $N \leq 9999$. 32.2 To use ordinal numbers.

*The first digit to the left of the decimal point indicates grade level; the second (and third), the major concept; and the digits to the right of the decimal point indicate the item number.

Major Concepts	Student Abilities
2. Number (cont.)	32.3 To use informally one-half, one-third, one-fourth, and two-thirds of a given unit. 32.4 To determine whether a given number is even or odd. 32.5 To interpret addition and subtraction on the number line.
	42.1 To determine the cardinality of any set where $N \geq 0$. 42.2 To use equivalence classes of fractions. 42.3 To determine prime numbers. 42.4 To determine the factors of a counting number. 42.5 To determine the least common multiple of two counting numbers. 42.6 To determine the greatest common factor of two counting numbers. 42.7 To determine "betweenness," "less than," "greater than," and "equal to" for whole numbers.
3. Numeration System (The emphasis will be on the decimal system.)	13.1 To write the basic numerals 0-9. 13.2 To write the numerals through 99. 13.3 To write various numerals for a given number, i.e., 1 + 4, 2 + 3, 3 + 2, 9 - 4, 5, etc. 13.4 To interpret place value through 99. 13.5 To use correctly the relational words ***greater than***, ***less than***, and ***equal to*** in connection with numbers through 99.
	23.1 To identify and write numerals through 999. 23.2 To write many symbols for the same number. 23.3 To interpret place value through 999. 23.4 To count by fives, tens, and hundreds. 23.5 To use the symbols =, <, and > in open sentences in connection with numbers through 999. 23.6 To write numerals in expanded notation.
	33.1 To write numerals through 9999. 33.2 To write many symbols for the same number. 33.3 To interpret place value through 9999. 33.4 To use the symbols =, <, and > in open sentences in connection with numbers through 9999. 33.5 To write numerals in expanded notation.
	43.1 To write all numerals. 43.2 To interpret place value for all numbers up to billions.
4. Operations and Properties (The basic operations will be developed via sets. Properties will be intuitively and inductively developed.)	14.1 To associate a + b with the union of two disjoint sets A and B, where a = N(A), b = N(B). 14.2 To use the commutative property in addition. 14.3 To associate a - b with the relative compliment of two disjoint sets A and B, where a = N(A), b = N(B). 14.4 To use the addition facts for two numbers, a and b, where $a + b \leq 10$. 14.5 To use the subtraction facts associated with 14.4.
	24.1 To use the identity element of addition. 24.2 To use the associative property in addition. 24.3 To represent the union of a sets with b in each set in the form a x b. 24.4 To use the commutative property in multiplication.

Major Concepts	Student Abilities
4. Operations and Properties (cont.)	24.5 To use the identity element of multiplication. 24.6 To represent the partition of one set into a sets with b elements in each set. 24.7 To use the inverse relationship between addition and subtraction. 24.8 To use the addition facts for two numbers, a and b, where $a + b \leq 18$. 24.9 To use the subtraction facts associated with 24.8. 24.10 To use the multiplication facts for two numbers, a and b, where $a \times b \leq 18$. 24.11 To use the division facts associated with 24.10.
	34.1 To use the associative property in multiplication. 34.2 To use the distributive property of multiplication over addition. 34.3 To use the inverse relationship between multiplication and division. 34.4 To use the multiplication facts for two numbers, a and b, where $a \times b \leq 45$. 34.5 To use the division facts associated with 34.4
	44.1 To use the multiplication facts for two numbers, a and b, where $a \times b \leq 100$. 44.2 To use the division facts associated with 44.1. 44.3 To use equivalence classes of fractions and some of their properties. 44.4 To identify a subset of the rationals which correspond to the whole numbers.
5. Computation	15.1 To use the basic facts $a + b = N$ where $a + b \leq 10$. 15.2 To compute sums of the form $a + b = N$ where $a + b \leq 99$. 15.3 To use the basic subtraction facts associated with 15.1. 15.4 To compute differences associated with 15.2.
	25.1 To use the basic facts $a + b = N$ where $a + b \leq 18$. 25.2 To compute sums of the form $a + b = N$ where $a + b \leq 999$. 25.3 To use the basic subtraction facts associated with 25.1. 25.4 To compute differences associated with 25.2.
	35.1 To compute sums of the form $a + b = N$ where $a + b \leq 9999$. 35.2 To compute differences associated with 35.1. 35.3 To use the basic facts $a \times b = N$ where $a \times b \leq 45$. 35.4 To use the basic division facts associated with 35.3. 35.5 To compute products of the form $a \times b = N$ where $a \times b \leq 9999$. Also, either $a \leq 10$ or $b \leq 10$. 35.6 To compute quotients associated with 35.5.
	45.1 To compute all products of the form $a \times b = N$. 45.2 To compute all quotients associated with 45.1.
6. Problem Solving (Emphasis will be on learning the relationship between the "physical" world and the "mathematical" world	16.1 To associate the equation forms $a + b = N$ and $a - b = N$ with physical situations. 16.2 To interpret the solution of an equation to the physical situation. 16.3 To "make up" physical problems which "fit" a given open sentence.

Major Concepts	Student Abilities
through use of sentences.)	26.1 To associate the equation forms a + N = b, N + a = b, a - N = b, and N - a = b with physical situations. 26.2 To process all equation forms introduced and interpret the results. 26.3 To "make up" physical problems which "fit" a given open sentence.
	36.1 To associate the equation forms a x b = N and a + b = N with appropriate physical situations. 36.2 To process all equation forms introduced and interpret the results. 36.3 To "make up" physical problems which "fit" a given open sentence. 36.4 To solve problems involving more than "one step."
	46.1 To process all equation forms introduced and interpret the results. 46.2 To "make up" physical problems which "fit" a given open sentence. 46.3 To solve problems involving more than operation.
7. Geometry (Intuitively develop some metric and non-metric concepts.)	17.1 To recognize points and lines. 17.2 To identify open and closed curves. 17.3 To determine whether points are inside, outside, or on closed curves. 17.4 To determine whether or not a point is between two other points. 17.5 To classify objects as spheres, cubes, cylinders, or rectangular solids.
	27.1 To recognize line segments. 27.2 To recognize quadrilaterals, triangles, squares, rectangles, and circles. 27.3 To draw lines, line segments, quadrilaterals, triangles, squares, rectangles, and circles. 27.4 To identify elementary properties of geometric forms in 17.5 and 27.2, e.g. congruency, number of sides, and number of vertices. 27.5 To use the concepts of interior and exterior with many of the geometric forms which have been introduced.
	37.1 To recognize and use rays, angles, and surfaces. 37.2 To work with intersections of lines and surfaces. 37.3 To identify and use some elementary properties of symmetry and similarity associated with the two-dimensional geometric forms introduced thus far. 37.4 To identify and use some of the elementary properties of the three-dimensional geometric forms introduced thus far. 37.5 To recognize two-dimensional regions. 37.6 To distinguish between regions and geometric forms in two dimensions.
	47.1 To recognize parallelism. 47.2 To use parallelism in two- and three-dimensional geometric forms. 47.3 To recognize perpendicularity.

Major Concepts	Student Abilities
7. Geometry (cont.)	47.4 To use perpendicularity in two- and three-dimensional geometric forms. 47.5 To recognize the perimeter of a region. 47.6 To recognize some polygons and polyhedrons which have not yet been introduced. 47.7 To recognize some of the projective properties in relation to the geometric ideas which have been introduced.
8. Measurement (Measurement will be the exact number of standard units. Consideration will be given to approximation.)	18.1 To use arbitrary units in linear measurement and capacity. 18.2 To use standard units in linear measurement (inch and foot) and capacity (pint and quart).
	28.1 To use arbitrary and standard units of length and capacity. 28.2 To use cup, pint, quart, and gallon in measuring situations.
	38.1 To use the standard units of inch and foot. 38.2 To use the common units of capacity (pint, quart, etc.). 38.3 To weigh objects using ounces and pounds.
	48.1 To use an arbitrary linear and two-dimensional measure as a standard unit. 48.2 To approximate the measure of the boundary and area of bounded plane geometrical forms.
9. Relations (Subsets of N x N)	39.1 To set up equivalence classes and use them informally to solve ratio problems.
	49.1 To use properties of equivalence classes to solve ratio problems.
10. Number Theory	310.1 To identify odd and even numbers. 310.2 To determine oddness or evenness of sums of odds or evens.
	410.1 To factor numbers in primes and composites. 410.2 To find common factors.
11. Miscellaneous Ideas (Non-mathematical ideas which are normally taught as part of the mathematics program.)	111.1 To identify penny, nickel, dime, quarter, and half-dollar and use these in making change for items costing less than one dollar. 111.2 To name the days of the week and tell time to the nearest hour.
	210.1 To identify penny, nickel, dime, quarter, half-dollar and dollar and use them in making change for items costing less than five dollars. 210.2 To name the months of the year and tell time to the nearest quarter hour.
	310.1 To identify all coins and paper money and use our money system. 310.2 To use a Fahrenheit thermometer. 310.3 To tell time.
	410.1 To discriminate among time zones.

THE DEVELOPMENTAL YEARS OF PIA

During the year 1965-66, the programs for Grades 1 and 3 were videotaped. During 1966-67, the programs for Grades 2 and 4 were videotaped. A developmental year for PIA had several principal features.

1. Based on the objectives of the program, each lesson was planned as to mathematical content, method, and television presentation.
2. Teachers within the viewing area of WHA-TV (channel 21) located in Madison, Wisconsin, were invited to participate through the usual channels of the Wisconsin School of the Air. From thirty to one hundred teachers per grade per year used the program during the developmental year.
3. Every four to eight weeks the teachers of a grade met at the R & D Center, or at the studios of WHA-TV, to give their evaluation of the programs and to hear a discussion by the staff of the Center of the key ideas to be presented in the next few weeks. On the basis of comments at these meetings, some programs and lesson sheets were changed or entirely rewritten.
4. Midyear and end-of-year achievement tests were given to all the children participating and the results were reported to the teachers.
5. At the end of the year the individual lessons were edited and bound as an exercise book for use during subsequent years.

PART TWO:
EVALUATION

III

EVALUATION SCHEME: SCOPE OF THE STUDY

During the 1965-66 school year Patterns in Arithmetic (PIA) Grades 1 and 3 were developed at the University of Wisconsin Research and Development Center. The TV tapes were aired over WHA-TV, the University of Wisconsin station, for the benefit of those teachers in and near Madison, Wisconsin, who agreed to participate in the developmental year program. Teacher and pupil materials were sent out periodically to participating teachers, and meetings were held regularly with teachers in order to obtain their reaction to the programs. Following this initial trial period the teacher notes and pupil exercises were modified and bound into separate workbook-type volumes.

Over the summer of 1966 arrangements were made with schools in Alabama and Wisconsin to participate in an extensive field study of PIA Grades 1 and 3. In enlisting classes to participate in the study, attention was given to two variables (beside state) that would likely be of interest to those persons making curriculum decisions. These variables were community size and socioeconomic status. The PIA staff wanted to gather information on how operational the program was with respect to community size and socioeconomic status in two rather different regional settings, Alabama and Wisconsin. Each community cooperating in the study was classified in one of four community size groups: community size 1 had a population range of 0–10,000; community size 2, 10,000–30,000; community size 3, 30,000–90,000; and community size 4, 90,000 and up. In addition, each class in community sizes 3 and 4 was classified into one of three socioeconomic groups: high, middle, or low. It was felt that it was not realistic to attempt a socioeconomic classification on schools in community sizes 1 and 2 since such schools are very likely quite heterogeneous. (See Figure 1.)

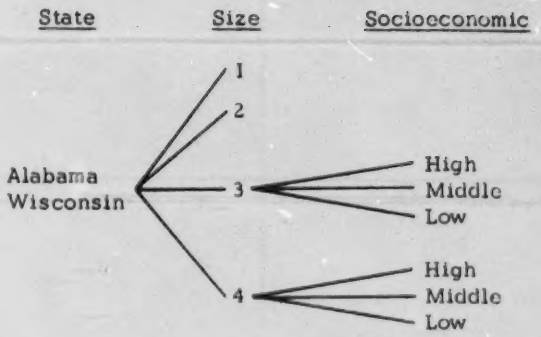

Figure 1. Classification Schema

The selection of appropriate evaluation instruments at each grade level was a major problem. It was decided that two types of tests were appropriate: (1) tests designed specifically to measure program achievement, i.e. the degree to which the objectives of PIA were accomplished, and (2) standard achievement tests covering both computation and concepts. The nature and construction of first type of test, developed for PIA by the Educational Testing Service, Princeton, New Jersey, will be discussed in the next section. Selection of standard achievement tests was based upon coverage of approximate computational skills and concepts and upon coverage of concepts consistent with the objectives of PIA.

After considerable debate, the following standard achievement tests were selected.

For Grade 1:
1. The *California Arithmetic Test, Lower Primary, Form W, Test 4.* This is a 45-item computation test published in 1957 by the California Test Bureau with 1963 norms.
2. The *ETS Cooperative Primary Tests, Mathematics, Form 12B.* This is a 55-item concepts test published in 1965 by Educational Testing Service.

For Grade 3:
1. The *Stanford Achievement Tests, Forms W and Y,* from the *Primary II Battery.* These are standard 60-item computa-

tion tests, copyrighted in 1965 by Harcourt, Brace and World.

2. The ***ETS Cooperative Primary Tests, Mathematics, Forms 23A, 23B.*** These are 60-item concepts tests, published in 1965 by Educational Testing Service.

Copies of each of the above tests may be obtained from the publishers. A sample of items from the various tests may be found in Appendix A. No pretest was given in Grade 1. In Grade 3 the ***Stanford Achievement Test, Form W,*** was used as a pretest and ***Form Y*** as a posttest for computation. The ***Cooperative Primary Test, Form 23B*** was used as a pretest and ***Form 23A*** as a posttest for concepts.

In order to minimize testing with a given class, it was decided to divide those classes participating at each grade level into two groups, group A and group B. Group A would take the computation tests and Group B would take the concepts tests. Moreover, both groups would take the achievement tests written specifically for the PIA program. Figure 2 identifies these groups more completely.

IV

THE PATTERNS IN ARITHMETIC ACHIEVEMENT TESTS

Educational Testing Service was requested to develop four tests to measure achievement relating to the PIA program, one for each semester at each grade level. The purpose of these tests was twofold: (1) to provide a measure of achievement during the 1966-67 field trial period, and (2) to be used as a part of the instructional package to future program users. Dr. Henry Van Engen provided the ETS staff with the general specifications (content and behavioral) under which the tests should be constructed. The tests were designed to be administered to classroom-sized groups and to be appropriately styled for the children involved. The tests were to be of sufficient length to be reliable, but not too long since the attention span of young children is short. The tests were designed to represent a cross-section of program content, with emphasis on major instructional objectives of the program. While most of the test items measure skills covered in the program, a few items attempt to measure the children's ability to extend their knowledge to somewhat unfamiliar settings.

Items for these four tests were based on pilot testing done during the previous year on the children who participated during the developmental stages (1965-66) of the program. ETS sent staff to Madison to view the TV tapes and study the program objectives before constructing the pilot tests.

The final forms for the two first-grade tests were to have 40 items with administration accomplished over two settings. The tests should be untimed with easy and difficult items intermixed. Each question would have three choices, the correct choice to be indicated by a large X. Questions would be read aloud by the teacher to minimize the verbal barrier and pace the class. Since each item would be read by the teacher, the child would have the opportunity to respond to every item. The final forms for the two third-grade tests were to be similar to the above except each test would have 45 items.

PRETESTING

Two pretest forms were constructed for each final form desired: two 25-item forms for Grade 1 and two 30-item forms for Grade 3. The margin of 10 and 15 items, respectively, over the number required in the final form served two purposes: first, it permitted the elimination of any bad items, and second, it allowed the item writers latitude for trying out various approaches to testing difficult concepts. Tables 1, 2, 5, and 6 were used to determine the number of items to be written in each content area. The Patterns In Arithmetic Teacher Guides and Pupil Exercise sheets were used to determine the main ideas presented under each topic and the methods of approach. Most items were written to measure pupils' understanding of ideas as they were presented in the program, but a number of items were also addressed to unique applications or extensions of program presentations.

The pretests were administered to a sample of Wisconsin schools that participated in the developmental year program. Staff at the R & D Center arranged for the administration of the pretests, but returned the completed test booklets to ETS for analysis. Each pretest form was submitted to the standard item analysis procedure at ETS. Following this analysis, final forms of the tests to be used in the field testing were constructed.

THE MIDYEAR TESTS: GRADES 1 AND 3

Tables 1 and 2 represent specifications furnished by the PIA staff to ETS for construction of the midyear tests. These outlines, apart from their use by ETS, reflect content emphasis in Grades 1 and 3. Pilot tests were constructed based in part upon these tables and from the piloted items a pool of items was developed from which the final forms were constructed.

Table 1. Content Outline for Midyear Examination, Grade 1

Percent of Items	Topic
10	Sets
	One-to-one correspondence, matching elements of equivalent sets
	Determining more than, less than
	Transitivity
	Position, grouping or size of elements of sets; non-effect on cardinality
40	Numbers
	Determining cardinality of sets
	Determining greater than, less than
	Betweenness of numbers
	Applications of "one-more-than"
20	Numeration Systems
	Recognizing and writing numerals
	Recognizing other than standard names for numerals—i.e., 2 + 3, 1 + 4, etc.
20	Operations, Properties, and Computation
	Recognizing addition situations
	Commutative property
	Addition facts ($a + b \leq 10$)
	Pictorial representation of addition
10	Geometry
	Concept of points, curves
	Open and closed curves
	Betweenness of points
	Points on same side of a curve, inside, outside, or on a curve

Table 2. Content Outline for Midyear Examination, Grade 3

Percent of Items	Topic
15	Numeration Systems
	Representation of numbers, writing numerals, recognizing and using non-standard names for numbers
	Place value, 2- and 3-digit numbers
	Grouping by tens (i.e., 29 + 3 becomes 3 tens and 2 over)
	Representation of number such as 9 hundreds, 9 tens, and 4 ones as 994
20	Numbers
	Comparison of numbers ($<$, $=$, $>$)
	Ordering numbers, betweenness of numbers
20	Properties
	Associative property of addition
	Commutative property of addition and of multiplication
	Distributive property
30	Operations and Computation
	Recognizing addition and subtraction situations and representing them in math sentences
	Recognizing multiplication and division situations and representing them in math sentences
	Recognizing comparative situations—how many more
	Relation of addition to subtraction, multiplication to division, addition to multiplication
	Addition and subtraction facts for $N \leq 9999$ ($N = a \pm b$)
	Multiplication facts for $N \leq 45$ ($N = a \times b$)
15	Geometry
	Open and closed curves; points inside and on curves
	Angles and angle terminology
	Triangles
	Polygons, 4 sided

20

ETS Analysis of the Final Midyear Test Forms. Grades 1 and 3

The two final forms of the midyear tests were administered in late January and early February, 1967, to classes in Alabama and Wisconsin participating in the field study. A random sample of about 4 tests from each class was sent to ETS for analysis. Analyses were performed on each test form (1) for Alabama, (2) for Wisconsin, and (3) for the two samples combined. Tables 3 and 4 summarize the test characteristics of the sample analyzed by ETS.

Table 3. Sample Statistics, PIA Midyear Test, Grade 1

Item No.	Alabama % Correct	Alabama r_{bis}	Wisconsin % Correct	Wisconsin r_{bis}	Combined % Correct	Combined r_{bis}
1	66	0.37	81	0.66	73	0.48
2	70	0.58	71	0.62	70	0.58
3	82	0.61	88	0.68	85	0.66
4	74	0.54	72	0.58	74	0.55
5	48	0.61	52	0.45	50	0.55
6	71	0.49	71	0.51	71	0.51
7	66	0.62	72	0.63	69	0.64
8	54	0.37	51	0.28	53	0.33
9	38	0.43	48	0.47	43	0.45
10	32	0.55	31	0.47	32	0.53
11	91	0.60	93	0.58	92	0.59
12	55	0.62	68	0.56	61	0.61
13	82	0.65	89	0.70	85	0.67
14	87	0.60	92	0.48	89	0.56
15	29	0.21	28	0.16	29	0.21
16	61	0.53	63	0.45	62	0.49
17	46	0.51	47	0.51	46	0.50
18	73	0.53	75	0.55	74	0.52
19	66	0.43	68	0.52	67	0.47
20	50	0.53	53	0.41	51	0.47
21	49	0.33	53	0.48	51	0.39
22	81	0.56	84	0.50	83	0.55
23	41	0.60	35	0.54	38	0.57
24	57	0.46	54	0.46	56	0.45
25	59	0.58	67	0.68	63	0.62
26	56	0.34	66	0.46	61	0.39
27	31	0.14	28	0.15	29	0.12
28	70	0.40	75	0.42	72	0.40
29	39	0.40	36	0.38	38	0.40
30	53	0.56	64	0.62	58	0.57
31	63	0.49	62	0.55	63	0.51
32	27	0.30	32	0.25	29	0.27
33	66	0.59	66	0.64	66	0.60
34	54	0.49	49	0.61	51	0.53
35	78	0.56	86	0.63	82	0.59
36	57	0.56	67	0.62	62	0.58
37	42	0.54	49	0.44	45	0.49
38	28	0.26	17	0.14	23	0.19
39	63	0.54	65	0.56	64	0.55
40	18	0.04	13	-0.02	15	0.02

		Sample Size	Mean	S.D.	Reliability
Summary:	Ala	365	22.5	7.21	.86
	Wis	310	23.8	6.46	.83
	Total	675	23.1	6.88	.85

Table 4. Sample Statistics, PIA Midyear Test, Grade 3

Item No.	Alabama % Correct	Alabama r_{bis}	Wisconsin % Correct	Wisconsin r_{bis}*	Combined % Correct	Combined r_{bis}
1	95	0.42	96	-	95	0.36
2	80	0.40	81	0.43	81	0.42
3	64	0.46	57	0.53	60	0.47
4	56	0.57	48	0.52	52	0.54
5	60	0.51	66	0.61	63	0.56
6	60	0.31	54	0.33	57	0.33
7	90	0.54	92	0.46	91	0.51
8	63	0.38	75	0.43	68	0.40
9	60	0.46	68	0.46	64	0.47
10	65	0.50	57	0.50	61	0.49
11	56	0.38	53	0.36	55	0.36
12	38	0.41	41	0.40	39	0.41
13	38	0.48	34	0.41	36	0.45
14	53	0.39	51	0.25	52	0.33
15	74	0.52	75	0.48	74	0.50
16	36	0.19	24	0.20	30	0.18
17	83	0.54	84	0.41	83	0.47
18	17	0.36	12	0.53	15	0.43
19	55	0.20	50	0.18	52	0.19
20	36	0.48	42	0.29	39	0.40
21	49	0.62	54	0.56	51	0.58
22	60	0.62	74	0.46	66	0.55
23	74	0.51	80	0.38	77	0.47
24	60	0.69	60	0.72	60	0.70
25	45	0.60	55	0.50	49	0.54
26	34	0.18	34	0.38	34	0.27
27	84	0.51	85	0.57	84	0.53
28	44	0.34	39	0.30	42	0.32
29	40	0.51	35	0.46	38	0.50
30	24	0.32	19	0.32	22	0.33
31	90	0.48	92	0.47	91	0.49
32	76	0.59	79	0.50	78	0.57
33	53	0.61	55	0.55	54	0.59
34	55	0.57	60	0.51	57	0.55
35	72	0.60	75	0.48	73	0.55
36	67	0.49	69	0.51	68	0.49
37	65	0.66	74	0.68	69	0.66
38	65	0.62	64	0.52	65	0.60
39	42	0.27	45	0.27	43	0.27
40	38	0.55	41	0.54	39	0.54
41	40	0.46	35	0.42	38	0.45
42	66	0.71	69	0.57	67	0.64
43	48	0.55	44	0.51	46	0.52
44	42	0.20	50	0.22	46	0.22
45	53	0.40	62	0.28	57	0.36

		Sample Size	Mean	S.D.	Reliability
Summary:	Ala	410	22.5	7.75	.86
	Wis	350	25.9	6.92	.83
	Total	760	25.7	7.38	.84

*Not reported on items more than 95% correct.

Discussion of Midyear Test Characteristics

ETS reported that although the tests were of somewhat greater than middle difficulty for their respective groups, they could be considered appropriate in difficulty for these groups. Mean scores were somewhat greater than half the number of items in each test, relatively few scores fell below the chance level, and there was negligible skewness in the distributions. The test reliabilities computed by the Kuder-Richardson Formula 20 were .85 and .84 for the Grade 1 and Grade 3 midyear tests, respectively.

THE END-OF-YEAR TESTS: GRADES 1 AND 3

Tables 5 and 6 represent specifications furnished by the PIA staff to ETS for construction of the PIA End-of-Year Tests. These outlines reflect program emphasis from midyear to end-of-year. As for the midyear test, pretest forms were constructed with the aid of the above outlines. These forms were piloted on children participating in the developmental year program, and the items used formed a pool of items from which the final forms were constructed.

ETS Analysis of the Final End-of-Year Forms, Grades 1 and 3

The two final forms were administered near the third week in May, 1967, to classes in Alabama and Wisconsin who took the midyear tests. A random sample of about 4 tests from each class was sent to ETS for analysis. Analyses were performed on each test form (1) for Alabama, (2) for Wisconsin, and (3) for the two samples combined. Tables 7 and 8 summarize the test characteristics of the sample analyzed by ETS.

Discussion of End-of-Year Test Characteristics

As for the midyear test, ETS reported that the tests were difficult, but appropriate for the two groups. The mean scores were somewhat greater than half the number of items in each test, relatively few scores fell below the chance level, and there was negligible skewness in the distributions. The test reliabilities computed by the Kuder-Richardson formula 20 were .81 and .87 for the Grade 1 and Grade 3 end-of-year tests, respectively.

Table 5. Content Outline for End-of-Year Examination, Grade 1

Percent of Items	Topic
40	Numbers
	Cardinality of sets of N elements ($N \leq 99$)
	Ordinality (of less than 5 objects)
	Betweenness
	One-more-than, one-less-than
	Ordering of numbers ≤ 99
	Zero—empty sets
	Counting by tens
15	Numeration Systems
	Writing numerals up to 99
	Recognizing nonstandard names for numbers
	Place value for numbers ≤ 99
	Using relational words ($<$, $>$, $=$) correctly
20	Operations, Computations
	Subtraction facts $a + b \leq 10$, computing differences
	Addition: $a + b \leq 99$
0	Money
	Values and equivalences of various coins
15	Problem Solving
	Recognizing subtraction and addition situations and writing sentences for the problems
	Interpreting numerical solutions to physical situations
	Making up problems that fit open sentences
10	Geometry
	Lines, points, open and closed curves, betweenness
	Common polygons—triangles, rectangles, squares
	Comparative thinking—longer, shorter
	Arbitrary and standard units of measurement—inches, feet

Table 6. Content Outline for End-of-Year Examination, Grade 3

Percent of Items	Topics
15	Numeration Systems
	Representation of numbers, writing numerals, recognition and use of nonstandard names for numbers
	Place value—4-digit numbers
	Regrouping for addition and subtraction
25	Numbers
	Comparison of numbers (<, =, >)
	Ordering numbers < 10,000, betweenness, ordinal numbers
	Role of zero in addition, subtraction, and multiplication
	Fractions
	Various representations for $\frac{1}{2}$, $\frac{1}{4}$, etc.
	Comparing fractions
	Number pairs
20	Properties
	Distributive law
	Associative law of multiplication
30	Operations and Computation
	Verbal problems
	Recognizing addition and subtraction situations and representing them in math sentences
	Recognizing multiplication and division situations and representing them in math sentences
	Associating the correct form of equations with appropriate physical situations
30	Operations and Computation (cont.)
	Relations of addition to subtraction, multiplication to division, multiplication to subtraction, etc.
	Addition and subtraction, $N = a \pm b$ ($N < 10,000$)
	Multiplication and division
	When one factor is a power or multiple of 10
	Horizontal multiplication, 1-digit by 3-digit
	Vertical multiplication, 1-digit by 2-digit
	Multiplication facts $a \times b = N$ ($N \leq 45$)
	Simple division problems
	Estimation
0	Money
	Currency, representing various amounts
	Counting money and making change
10	Geometry
	Congruent and similar triangles, polygons
	Circles, circle terminology
	Linear measurement
	Inches, feet, yards and conversions from one unit to another
	Approximation of length
	Capacity measure
	Concept of perimeter of figures
	Polygons

Table 7. Sample Statistics, PIA End-of-Year Test, Grade 1

Item No.	Alabama % Correct	Alabama r_{bis} *	Wisconsin % Correct	Wisconsin r_{bis}	Combined % Correct	Combined r_{bis}
1	92	0.56	92	0.58	92	0.57
2	96	-	92	0.47	94	0.44
3	77	0.57	76	0.64	77	0.60
4	66	0.55	55	0.55	62	0.55
5	63	0.62	72	0.57	67	0.59
6	83	0.61	84	0.58	83	0.57
7	64	0.57	67	0.55	65	0.55
8	62	0.44	57	0.46	59	0.43
9	51	0.20	70	0.29	60	0.24
10	66	0.61	62	0.54	65	0.57
11	47	0.54	44	0.44	46	0.50
12	59	0.56	58	0.45	59	0.50
13	45	0.42	48	0.36	47	0.39
14	73	0.49	77	0.60	74	0.51
15	55	0.14	60	0.32	57	0.22
16	49	0.22	52	0.44	50	0.30
17	46	0.26	54	0.51	50	0.37
18	75	0.45	79	0.47	77	0.46
19	43	0.48	39	0.46	41	0.47
20	49	0.52	54	0.48	51	0.49
21	89	0.49	90	0.51	90	0.51
22	80	0.67	85	0.57	82	0.61
23	55	0.31	62	0.34	58	0.31
24	52	0.17	44	0.43	49	0.29
25	66	0.66	66	0.58	66	0.61
26	42	0.47	36	0.53	39	0.49
27	32	0.64	31	0.43	31	0.53
28	75	0.35	80	0.55	77	0.42
29	69	0.52	74	0.44	71	0.48
30	56	0.43	57	0.33	57	0.38
31	28	0.36	25	0.52	26	0.42
32	37	0.40	27	0.30	32	0.33
33	59	0.53	56	0.42	58	0.48
34	59	0.61	55	0.50	58	0.56
35	52	0.45	48	0.58	51	0.51
36	54	0.34	49	0.35	52	0.35
37	35	0.43	37	0.38	36	0.41
38	67	0.48	65	0.42	66	0.44
39	47	0.51	40	0.33	44	0.42
40	45	0.24	42	0.36	44	0.29

		Sample Size	Mean	S.D.	Reliability
Summary:	Ala	310	23.6	6.37	.82
	Wis	240	23.6	6.48	.81
	Total	550	23.6	6.40	.81

*Not reported on items more than 95% correct.

Table 8. Sample Statistics, PIA End-of-Year Test, Grade 3

Item No.	Alabama % Correct	Alabama r_{bis}	Wisconsin % Correct	Wisconsin r_{bis}	Combined % Correct	Combined r_{bis}
1	82	0.62	87	0.60	84	0.62
2	69	0.48	76	0.53	73	0.52
3	75	0.70	83	0.49	79	0.61
4	86	0.51	89	0.48	88	0.53
5	54	0.56	61	0.51	57	0.54
6	81	0.51	79	0.40	80	0.45
7	63	0.44	54	0.40	58	0.41
8	65	0.41	64	0.34	64	0.36
9	61	0.36	62	0.44	62	0.39
10	81	0.48	88	0.52	84	0.49
11	81	0.63	86	0.55	83	0.57
12	53	0.66	58	0.60	55	0.63
13	46	0.62	51	0.62	48	0.61
14	60	0.31	64	0.19	62	0.26
15	58	0.62	64	0.52	61	0.58
16	72	0.34	75	0.52	74	0.41
17	78	0.46	80	0.47	79	0.47
18	48	0.43	41	0.54	44	0.47
19	69	0.49	73	0.45	71	0.48
20	64	0.23	57	0.37	60	0.28
21	48	0.55	41	0.62	44	0.57
22	36	0.36	35	0.27	35	0.31
23	48	0.43	50	0.49	49	0.46
24	32	0.23	40	0.29	36	0.26
25	60	0.42	59	0.42	59	0.42
26	62	0.42	54	0.57	59	0.48
27	42	0.51	42	0.68	42	0.57
28	32	0.50	36	0.43	34	0.47
29	44	0.40	45	0.47	44	0.42
30	36	0.11	32	0.29	33	0.18
31	86	0.62	92	0.52	89	0.61
32	79	0.55	83	0.48	81	0.52
33	78	0.68	78	0.60	78	0.64
34	74	0.57	78	0.48	76	0.54
35	67	0.51	66	0.47	66	0.49
36	52	0.57	60	0.52	56	0.55
37	71	0.51	76	0.45	73	0.48
38	76	0.69	82	0.64	79	0.68
39	69	0.66	72	0.55	70	0.61
40	71	0.65	63	0.54	67	0.58
41	64	0.74	68	0.78	66	0.76
42	34	0.40	33	0.40	34	0.40
43	64	0.56	61	0.37	63	0.52
44	51	0.56	51	0.55	51	0.55
45	36	0.49	35	0.44	36	0.47

		Sample Size	Mean	S.D.	Reliability
Summary:	Ala	385	27.5	8.15	.88
	Wis	355	28.2	7.45	.85
	Total	740	27.8	7.82	.87

V

THE TESTING SEQUENCE AND DESCRIPTIVE STATISTICS

Classes in the first- and third-grade study were randomly divided into two groups, group A and group B. Both groups took the achievement tests written for PIA by Educational Testing Service (ETS). In addition, group A took the California test in the first grade at the end of the year and group B took the ETS, Form 12B, test at the end of the year. In Grade 3, group A took the Stanford, Form W, test (pretest) and the Stanford, Form Y, test (posttest); group B took the ETS, Form 23B, test (pretest) and the ETS, Form 23A, test (posttest). Thus, for the third grade, it was possible to obtain a measure of growth in both computation and concepts on standard achievement tests. Figure 2 outlines the testing sequence. The tests are ordered for each group as they were given during the 1966-67 field testing period. Groups A and B contain classes in both Alabama and Wisconsin. Specific details as to the number of children taking each test and the number of classes from each state are given below.

For each test given in the first- and third-grade study the following descriptive statistics are reported for Alabama, Wisconsin, and the two combined: the test mean, the test standard deviation, an ordered distribution of all class means for the test, and a frequency distribution of test scores. Where appropriate, norms are given and the PIA group compared with the norms.

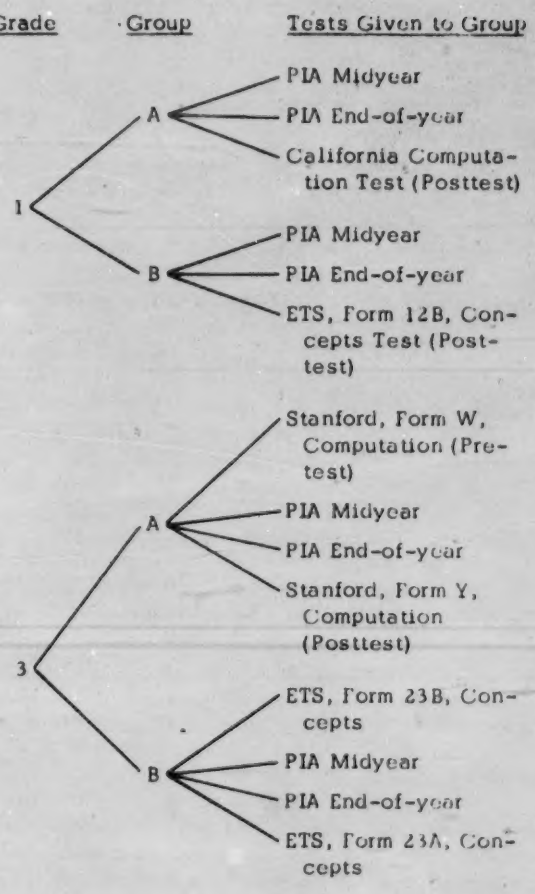

Figure 2. Testing Schema

TESTING: GRADE 1

Report 1: Patterns in Arithmetic Midyear Test

A midyear test was developed especially for the PIA program by Educational Testing Service, Princeton, New Jersey. Before the PIA program was developed certain concepts and ideas deemed important for the first grade were decided upon by the staff at the R & D Center. The PIA Midyear Test is an attempt to measure just how successful the program is in teaching these concepts up to midyear.

Item statistics for the Alabama and Wisconsin groups were very similar (see Appendix B). Items that were difficult in Wisconsin were difficult in Alabama and vice versa.

There are several aspects stressed in the Patterns first-grade program. Some of these aspects, and the associated problems from the midyear test designed to measure achievement in them, are discussed below.

(1) The idea that a given number has many different names is stressed often in the Patterns program. For example, 5 + 3 and 3 + 5 are other names for 8. Problems 10, 16, 18, 22, and 39 attempted to measure how flexible the children were in recognizing different names for a given number. The difficulty of these items for Alabama and Wisconsin indicates that the children have a good grasp of this concept.

(2) Terminology such as *greater than* and *less than* also seem to be understood by most children. Problems 1 and 7 test these terms.

(3) The commutative law of addition was recognized by most children, even when it occurred in a physical situation. Problems 2, 13, and 14 measure skills related to the commutative law.

(4) Another key point in the Patterns program is the *use* of the physical situation (or model) to represent addition. Results from Problems 22 and 24 show that this idea was not understood by midyear. However, as will be pointed out later, the pupils responded differently by the end of the year.

(5) Perhaps the most surprising finding from the midyear test was the poor performance by most children on problems involving geometrical ideas. Ideas such as open and closed curves, betweenness, and inside and outside of a closed curve were covered during the first semester. On the 40-item test there were 6 items involving geometrical ideas, Problems 8, 15, 20, 26, 30, and 40. Only one item was answered correctly over 50% of the time. Since each item had only three choices one would expect 33% correct by random guessing. These results seem to imply that several of the geometrical concepts which we are trying to teach first graders are beyond their immediate grasp. This, however, does not necessarily mean that we should drop teaching these geometrical ideas to first graders, for we all know that there are certain ideas that take quite a while to "sink in." Becoming familiar with a new idea is often prerequisite to attaining the concept at point. In fact, many teachers felt the children knew much more about geometry than they were able to show on the test.

Generally the test results were satisfactory. It is easy to guess which incorrect answer a first grader would pick for the question "What number must you add to the number two to get the next counting number?" and the choices are 1, 2, and 3. 35% got this item correct.

The PIA Grade 1 Midyear Test was administered to 92 classes in Alabama and 81 classes in Wisconsin near the end of January, 1967. Out of a total of 4410 pupils who took the test, 2476 were from Alabama classes and 1934 were from Wisconsin classes. Table 9 summarizes the results of the 40-item PIA test. The ordered distribution of 173 class means is given in Table 10. Some class means were disappointingly low. Not only was there considerable variation among class means, but there was also large within-class variance for many classes, particularly in Alabama.

There are no norms for the test. The frequency distribution of test scores for Alabama and Wisconsin is given in Table 11.

Table 9. PIA Midyear Test Summary

	Alabama	Wisconsin	Total
Number of Pupils	2476	1934	4410
Mean	21.4	22.8	22.0
Standard Deviation	7.17	6.15	6.78

Table 10. Distribution of Class Means for PIA Midyear Test, Grade 1

10.6	18.1	20.7	22.7	23.9	25.4
11.2	18.1	20.7	22.7	24.0	25.6
12.3	18.2	21.0	22.7	24.0	25.9
12.7	18.3	21.0	22.7	24.0	26.0
13.3	18.5	21.0	22.7	24.2	26.1
13.7	18.6	21.3	22.8	24.3	26.2
14.3	18.7	21.3	22.8	24.3	26.3
14.6	18.8	21.4	22.9	24.4	26.5
15.0	19.1	21.5	22.9	24.5	26.5
15.3	19.2	21.5	23.0	24.5	26.6
15.3	19.2	21.6	23.0	24.5	26.7
15.5	19.2	21.7	23.0	24.5	26.8
15.7	19.2	21.8	23.1	24.5	26.8
15.8	19.3	21.9	23.1	24.5	26.9
15.9	19.3	22.1	23.1	24.5	27.5
16.0	19.3	22.1	23.1	24.6	27.7
16.0	19.5	22.1	23.1	24.6	27.8
16.2	19.7	22.1	23.2	24.7	27.8
16.2	19.7	22.2	23.3	24.7	28.0
16.2	19.7	22.3	23.3	24.7	28.3
16.3	19.9	22.3	23.3	24.9	28.6
16.5	19.9	22.5	23.4	25.0	28.8
16.5	20.0	22.5	23.4	25.0	28.8
16.7	20.1	22.5	23.5	25.1	28.8
17.4	20.3	22.6	23.5	25.1	30.6
17.6	20.3	22.6	23.5	25.1	31.7
18.0	20.5	22.6	23.6	25.2	31.9
18.0	20.5	22.6	23.7	25.2	32.0
18.0	20.5	22.6	23.9	25.4	

Table 11. Frequency Distribution of Scores on PIA Midyear Test, 40 Items, Grade 1

Score Interval	Frequency		Percent		Percent Below	
	Ala.	Wis.	Ala.	Wis.	Ala.	Wis.
39-40	1	1	.1	.1	99.9	99.9
37-38	21	2	.8	.1	99.1	99.8
35-36	32	29	1.3	1.5	97.8	98.3
33-34	75	70	3.0	3.6	94.8	94.7
32	53	57	2.1	2.9	92.7	91.8
31	72	58	2.9	3.0	89.8	88.8
30	80	77	3.2	4.0	86.6	84.8
29	79	88	3.2	4.6	83.4	80.2
28	102	93	4.1	4.8	79.3	75.4
27	105	111	4.2	5.7	75.1	69.7
26	125	104	5.0	5.4	70.1	64.3
25	134	87	5.4	4.5	64.7	59.8
24	130	112	5.2	5.8	59.5	54.0
23	129	103	5.2	5.3	54.3	48.7
22	139	105	5.6	5.4	48.7	43.3
21	112	112	4.5	5.8	44.2	37.6
20	118	123	4.8	6.4	39.4	31.2
19	111	118	4.5	6.1	34.9	25.1
18	111	89	4.5	4.6	30.4	20.5
17	112	83	4.5	4.3	25.9	16.2
16	104	78	4.2	4.0	21.7	12.2
15	98	56	4.0	2.9	17.6	9.3
14	78	38	3.2	1.9	14.4	7.4
13	76	47	3.1	2.4	11.3	5.0
12	63	35	2.5	1.8	8.8	3.2
10-11	83	32	3.4	1.7	5.4	1.5
8-9	62	15	2.5	.8	2.9	.7
6-7	28	6	1.1	.3	1.8	.4
4-5	14	3	.6	.2	1.2	.2
2-3	15	1	.6	.1	.6	.1
0-1	14	1	.6	.1	.0	.0

Report 2: Patterns in Arithmetic End-of-Year Test

An end-of-year test was developed especially for the PIA program by Educational Testing Service. Before the PIA program was developed certain concepts and ideas deemed important for the first grade were decided upon by the staff at the R & D Center. The PIA End-of-Year Test is an attempt to measure just how successful the program is in teaching these concepts. The PIA Midyear Test covered concepts presented during the first half of the year while the End-of-Year Test placed emphasis on content from midyear on.

For the most part item difficulties for Alabama and Wisconsin were about the same (see Appendix B). The items on the End-of-Year Test covered several important content areas emphasized by the Patterns Program. Some of these areas, and the associated problems from the End-of-Year Test designed to measure achievement in them, are discussed below.

1) Considerable attention was devoted to developing the ability to write the appropriate number sentence for a given problem such as "John has 3 apples and buys some more. Then he has 7 apples." The sentence that represents the story of the problem is, then, $3 + \square = 7$. Problems 24 and 26 test this ability. Both of these problems were very difficult for the children.

2) The concept of numeration includes several abilities: writing numerals, recognizing nonstandard names for numerals, place value, and ordering. Problems 1, 8, 10, 20, 25, 27, 31, 34, and 40 test this important concept. Numeration is a very difficult concept to test in the first grade and for the most part the results on these items are encouraging.

3) In general the children did well on terminology and on problems requiring a knowledge of number facts. Problems 4, 5, 6, 30, 33, 35, 2, 21, 22, and 29 fit one of these categories.

4) On the PIA Midyear test the children had trouble recognizing the addition sentence to fit a physical model. However, by the end of the year they did very well in recognizing the sentence to fit a subtraction model (Problem 3) One of the objectives of PIA is to develop a referent for addition and subtraction to make these operations meaningful.

The PIA Grade 1 End-of-Year Test was administered to 165 classes, 89 in Alabama and 76 in Wisconsin near the end of May, 1967. Out of a total of 4150 pupils who took the test, 2260 were from Alabama classes and 1890 were from Wisconsin classes. Table 12 summarizes the results of the 40-item PIA End-of-Year Test.

The ordered distribution of 165 class means is given in Table 13. Note that the range of class means is from 12.9 to 36.3 on the 40-item test. Not only was there considerable variation among class means, but there was also large within-class variance, particularly in Alabama. It is not surprising that there were large differences among classes, since the study was done across socioeconomic groups in both Alabama and Wisconsin.

There are no norms for the test. The frequency distribution of scores for Alabama and Wisconsin is given in Table 14.

Report 3: The California Achievement Test

To obtain a measure of computation proficiency at the end of Grade 1, *Test 4: Arithmetic Fundamentals* from the *California Arithmetic Test, Lower Primary, Form W* was used. The test consists of two sections, C and D. Section C is a 25-item addition test containing items such as $\begin{array}{r} 3 \\ +\ 6 \end{array}$ and $\begin{array}{r} 9 \\ +\ 0 \end{array}$ and a few more complicated problems such as $\begin{array}{r} 25 \\ +\ 30 \end{array}$. Section D is a 20-item subtraction test with problems similar to those in Section C. It should be noted that the problem format adhered to throughout the Patterns in Arithmetic Grade 1 program was 3 + 6 = ___ rather than the above format used on the California Test. It is likely that this difference in format had little effect on the group mean.

The California Arithmetic Test was administered to 81 classes in Wisconsin and Alabama. Out of a total of 2058 pupils who took the test, 1020 were from Wisconsin classes and 1037 were from Alabama classes. Table 15 summarizes the results. The 25-item addition test and 20-item subtraction test are considered as one 45-item test. The ordered distribution of 81 class means given in Table 16 indicates considerable variance among the class means. The California Arithmetic Test is not difficult, and it is not surprising that several class means were quite high. For many classes the standard deviation was high and it was evident from the class frequency distributions that many of the teachers, particularly in Alabama, were faced with a wide range of pupil abilities. In fact, the range for some Alabama classes was over 40 on a test which contained only 45 items.

Table 12. PIA End-of-Year Test Summary

	Alabama	Wisconsin	Total
Number of Pupils	2260	1890	4150
Mean	23.3	23.5	23.4
Standard Deviation	6.42	6.19	6.32

Table 13. Distribution of Class Means for PIA End-of-Year Test, Grade 1

12.9	19.8	21.8	23.2	25.2	27.1
13.7	19.9	21.9	23.2	25.2	27.2
15.1	19.9	22.0	23.2	25.3	27.5
15.5	20.0	22.0	23.2	25.3	27.6
15.6	20.1	22.0	23.3	25.3	27.7
15.9	20.2	22.0	23.3	25.4	28.0
16.0	20.2	22.1	23.3	25.5	28.0
16.7	20.2	22.1	23.4	25.5	28.0
17.1	20.3	22.1	23.5	25.5	28.2
17.3	20.4	22.2	23.5	25.6	28.4
17.3	20.5	22.3	23.6	25.6	28.5
17.4	20.6	22.3	23.7	25.6	28.6
18.1	20.8	22.3	23.8	25.7	28.6
18.2	20.8	22.3	23.9	25.8	29.7
18.2	20.8	22.4	24.0	25.8	29.7
18.5	20.9	22.4	24.0	25.9	30.1
18.6	21.1	22.5	24.0	25.9	30.2
18.6	21.1	22.6	24.1	25.9	30.5
18.6	21.3	22.6	24.2	26.0	30.6
18.7	21.3	22.7	24.2	26.2	30.6
19.1	21.4	22.7	24.2	26.2	30.9
19.2	21.4	22.8	24.4	26.3	31.3
19.2	21.4	22.8	24.8	26.3	31.3
19.4	21.4	22.9	24.8	26.4	32.5
19.6	21.4	23.0	24.8	26.4	34.0
19.6	21.5	23.0	25.0	26.6	34.2
19.8	21.7	23.1	25.1	26.9	36.3
19.8	21.7	23.1			

Table 14. Frequency Distribution of Scores on PIA, End-of-year Test, 40 Items, Grade 1

Score Interval	Frequency		Percent		Percent Below	
	Ala.	Wis.	Ala.	Wis.	Ala.	Wis.
39-40	10	6	.4	.3	99.6	99.7
37-38	32	25	1.4	1.3	98.2	98.4
35-36	63	49	2.8	2.6	95.4	95.8
34	43	29	1.9	1.5	93.5	94.3
33	59	47	2.6	2.5	90.9	91.8
32	50	51	2.0	2.7	88.7	89.1
31	67	52	3.0	2.8	85.7	86.3
30	82	78	3.6	4.1	82.1	82.2
29	84	95	3.7	5.0	78.2	77.2
28	98	93	4.3	4.9	73.9	72.3
27	110	86	4.8	4.5	69.2	67.8
26	118	102	5.2	5.4	63.9	62.4
25	127	96	5.6	5.1	52.4	51.6
23	116	121	5.1	6.4	47.3	45.2
22	129	119	5.7	6.3	41.6	38.9
21	122	99	5.4	5.2	36.4	33.7
20	123	104	5.4	5.5	30.7	28.2
19	123	101	5.4	5.3	25.2	22.9
18	127	92	5.6	4.9	19.6	18.0
17	114	85	5.0	4.5	14.6	13.5
16	85	70	3.7	3.7	10.9	9.8
15	68	45	3.0	2.4	7.9	7.4
14	39	51	1.7	2.7	6.2	4.7
13	47	33	2.1	1.7	4.1	3.0
12	39	24	1.7	1.3	2.4	1.7
10-11	33	16	1.4	.8	1.0	.9
8-9	11	10	.5	.5	.5	.3
6-7	4	3	.2	.2	.3	.1
4-5	1	1	.1	.1	.2	.0
2-3	1	0	.1	.0	.1	.0
0-1	1	0	.1	.0	.0	.0

Table 15. California Arithmetic Test Summary

	Alabama	Wisconsin	Total
Number of Pupils	1037	1021	2058
Mean	30.8	33.9	32.3
Standard Deviation	11.36	7.56	9.78

Table 16. Distribution of Class Means on the California Test, Grade 1

14.9	28.9	31.2	33.5	36.4	38.0
15.1	29.0	31.2	33.6	36.4	38.0
15.1	29.0	31.3	34.0	36.5	38.2
15.8	29.4	31.4	34.5	36.6	38.3
18.7	29.6	32.0	34.8	36.6	38.4
22.3	29.8	32.0	35.0	36.6	38.7
23.3	29.8	32.5	35.2	36.7	38.7
24.5	30.1	32.5	35.2	36.7	38.9
26.0	30.5	32.7	35.7	36.8	39.0
27.0	30.8	32.8	36.0	37.3	39.6
28.3	30.9	32.9	36.0	37.4	40.6
28.4	31.0	33.0	36.1	37.4	41.4
28.5	31.0	33.2	36.3	37.9	41.8
28.7	31.1	33.4			

Table 17. Frequency Distribution of Scores, California Test, Grade 1

Score Interval	Frequency		Percent		Percent Below	
	Ala.	Wis.	Ala.	Wis.	Ala.	Wis.
44-45	17	16	1.7	1.6	98.3	98.4
42-43	106	84	10.2	8.2	88.1	90.2
40-41	107	124	10.3	12.1	77.8	78.1
38-39	134	134	12.9	13.1	64.9	65.0
36-37	110	163	10.6	16.0	54.3	49.0
34-35	114	145	11.0	14.2	43.3	34.9
32-33	69	86	6.6	8.4	36.7	26.5
30-31	57	64	5.5	6.3	31.2	20.2
28-29	36	41	3.5	4.0	27.7	16.2
26-27	22	38	2.1	3.7	25.6	12.5
24-25	30	37	2.9	3.6	22.7	8.9
22-23	28	17	2.7	1.7	20.0	7.2
20-21	26	18	2.5	1.8	17.5	5.4
18-19	21	10	2.0	1.0	15.5	4.4
16-17	25	7	2.4	.7	13.1	3.7
14-15	12	7	1.2	.7	11.9	3.0
12-13	19	9	1.8	.9	10.1	2.1
10-11	12	2	1.2	.2	8.9	1.9
8-9	20	6	1.9	.6	7.0	1.3
6-7	13	4	1.3	.4	5.7	.9
4-5	21	3	2.0	.3	3.7	.6
2-3	27	4	2.6	.4	1.1	.2
0-1	11	2	1.1	.2	.0	.0

Table 18. Norms for the California Test, Grade 1

Raw Score	Percentile Rank	Raw Score	Percentile Rank
42-45	99	30-31	46
41	98	29	42
40	95	27-28	38
39	90	26	34
38	84	24-25	30
37	76	23	27
36	70	21-22	24
35	62	15-16	14
34	58	9-10	7
33	54	3	2
32	50	1	1

Note—For a raw score of 33 the corresponding percentile rank is 54. This means that 33 is the score, below which 54% of the scores lie.

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The frequency distribution of test scores for Alabama and Wisconsin is given in Table 17. Table 18 gives norms for the California Test and may be used in conjunction with Table 17 in interpreting results. From the norms Table 18, the 50th percentile score is 32. This means that 50% of the norms group scored below 32. From Table 17, 31.2% in Alabama and 20.2% in Wisconsin scored below 32. Clearly the Patterns children did much better than the norm group.

Report 4: The ETS Cooperative Primary Test, Form 12B

The ETS Cooperative Primary Test—Mathematics, Form 12B was given to approximately half of the classes participating in the Patterns in Arithmetic field study in the first grade. The test was given near the end of the 1966-67 school year to both Alabama and Wisconsin classes. The *Form 12B* test is a test of concepts rather than computation proficiency. There are, however, a few problems on the test which measure number facts. Spring norms for the test are included in this report.

The ETS Cooperative Primary Test, Form 12B was administered to 80 first grade classes, 43 in Alabama and 37 in Wisconsin. Out of a total of 2120 pupils who took this test, 1159 were from Alabama classes and 961 were from Wisconsin classes. Table 19 summarizes the results on the 55-item test.

Table 19. ETS, Form 12B, Summary

	Alabama	Wisconsin	Total
Number of Pupils	1159	961	2120
Mean	38.4	38.7	38.6
Standard Deviation	8.68	7.37	8.11

The ordered distribution of 80 class means is given in Table 20. Again, the class means show considerable variance. The within-state standard deviation is large, but not as large as on the California computation test. For several classes in Alabama the range of scores was quite large.

Table 20. Distribution of Class Means on the ETS, Form 12B, Test

24.4	38.9
25.7	39.0
29.3	39.3
29.4	39.6
29.5	39.6
30.0	39.6
30.3	39.7
31.2	39.8
32.6	39.9
32.6	39.9
32.7	40.1
32.8	40.1
33.3	40.2
33.8	40.5
33.9	40.6
34.3	40.9
34.4	41.0
34.5	41.1
35.1	41.1
35.2	41.2
35.2	41.2
35.4	42.3
35.5	42.5
35.9	42.7
36.1	42.9
36.7	43.1
36.7	43.2
36.8	43.3
36.9	43.5
37.3	43.6
37.4	44.2
37.5	44.8
37.5	44.9
37.5	45.3
37.9	45.5
37.9	45.7
38.0	48.4
38.4	48.9
38.4	50.6
38.8	53.1

The frequency distribution of test scores is given in Table 21. Table 22 gives national norms for the ETS, Form 12B, test as reported in 1967 by Educational Testing Service and may be used in conjunction with Tables 20 and 21 to interpret results. Note that from Table 19 the mean for Alabama is 38.4 and for Wisconsin it is 38.7. Both compare favorably with the norms mean of 35.3. From Table 22, 74% of the norms group scored below 41. From Table 21, only 56.5% in Alabama and 55.1% in Wisconsin scored below 41.

Table 21. Frequency Distribution of Scores, ETS, Form 12B

Score Interval	Frequency		Percent		Percent Below	
	Ala.	Wis.	Ala.	Wis.	Ala.	Wis.
55	12	1	1.0	.1	99.0	99.9
53-54	25	5	2.2	.5	96.8	99.4
51-52	45	24	3.9	2.5	92.9	96.9
49-50	58	42	5.0	4.4	87.9	92.5
47-48	74	65	6.4	6.8	81.5	85.7
45-46	84	80	7.2	8.3	74.3	77.4
43-44	104	106	9.0	11.0	65.3	66.4
41-42	102	109	8.8	11.3	56.5	55.1
39-40	122	103	10.5	10.7	46.0	44.4
37-38	91	87	7.9	9.1	38.1	35.3
35-36	94	75	8.1	7.8	30.0	27.5
33-34	71	73	6.1	7.6	24.0	19.9
31-32	58	54	5.0	5.6	18.9	14.3
29-30	55	36	4.7	3.7	14.2	10.6
27-28	44	39	3.8	4.1	10.4	6.5
25-26	38	19	3.3	2.0	7.1	4.5
23-24	27	18	2.3	1.9	4.8	2.6
21-22	18	16	1.6	1.7	3.2	.9
19-20	18	5	1.6	.5	1.6	.4
17-18	6	1	.5	.1	1.1	.3
15-16	5	1	.4	.1	.7	.2
13-14	7	0	.6	.0	.1	.2
11-12	0	1	.0	.1	.1	.1
9-10	1	1	.1	.1	.0	.0
0-8	0	0	.0	.0	.0	.0

Table 22. ETS National Norms: Grade 1 (Spring)

Raw Score Form 12B	Mid-Percentile Rank
55	
54	
	99.9
53	99.9
	99.7
52	99.7
	99
51	99
	98
	98
49-50	98
	97
47-48	95
46	93
45	91
44	87
42-43	81
41	77
39-40	68
37-38	57
34-36	42
31-33	34
27-30	20
22-26	9
18-21	2
16-17	1
13-15	0.8
11-12	0.5
9-10	0.3
7-8	0.3
5-6	0.2
3-4	0.1
1-2	0.1
0	0.1
35.3 Mean	
8.4 Standard Deviation	
41.0 Upper Quartile	
35.6 Median	
29.5 Lower Quartile	

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TESTING: GRADE 3

Report 5: Patterns in Arithmetic Midyear Test, Grade 3

A midyear test was developed especially for the PIA program by Educational Testing Service. Before the PIA program was developed certain concepts and ideas deemed important for the third grade were decided upon by the staff at the R & D Center. The PIA Midyear Test is an attempt to measure just how successful the program is in teaching these concepts up to midyear.

Item difficulties for Alabama and Wisconsin were very similar (see Appendix B). The items on the Midyear Test covered several important content areas emphasized by the Patterns program. Some of these areas, and the associated problems from the Midyear Test designed to measure them, are discussed below.

1) As in PIA Grade 1, geometry receives considerable attention. Questions 1, 6, 22, and 26 are devoted to geometry, but only Question 22 is at the terminology level. Questions 1 and 22 both relate to the square, but from the item difficulties it is apparent that Question 1 was much easier. Question 22 merely asks which figure is a square. Evidently, the unstable position of the square was a source of confusion. Question 6 involved the concept of an open curve; this item was difficult in both Wisconsin and Alabama. Problem 26 involves a complicated application of the term "right angle." This problem was extremely difficult.

2) Questions 7 and 23 test whether the children have developed a referent for addition and multiplication. From the item difficulties it appears that the children have indeed developed a referent for those two operations.

3) Problems 39 and 43 test the distributive property. Both of these problems were missed frequently.

4) Problems 34 and 35 involve recognizing the appropriate way to name a pictorial array of objects. This idea was covered in the PIA program, and relative to the difficulty of the items, the results are encouraging.

5) The concept of numeration is very difficult to teach and, because it has so many aspects, it is also difficult to test. Aspects of numeration include writing numerals, recognizing nonstandard names for numerals, place value, and ordering. Problems 2, 10, 11, 18, 24, 25, and 33 test various aspects of numeration. Most of these problems were very difficult and are best understood by looking at them individually along with their associated item difficulties for Alabama and Wisconsin.

Many items on this test require a rather sophisticated style of problem solving for third graders, and few of the problems measure isolated terminology and number facts, but more complicated mental processes.

One curious aspect of the testing is worth mentioning. Problem 6 which tests the distinction between open and closed curves, is the same as Problem 26 on the PIA Midyear Test for Grade 1. Looking at the item difficulties one finds the item somewhat easier for first graders in Wisconsin than third graders and about the same difficulty for both grades in Alabama. The explanation for this is that in PIA Grade 1 the material is covered for the first time and the distinction between open and closed curves is put in simple but precise terms. The PIA Grade 3 programs represents a second spiral through this topic and coverage is not at the same level as in Grade 1. It is important to keep in mind that children in the PIA Grade 3 study had not had PIA Grade 1.

The PIA, Grade 3, Midyear Test was administered to 174 classes in Alabama and Wisconsin (92 Alabama classes) near the end of January, 1967. Out of a total of 4770 pupils who took the test 2630 were from Alabama classes and 2140 were from Wisconsin classes. Table 23 summarizes the results of the 45-item PIA Midyear Test.

Table 23. PIA Midyear Test Summary, Grade 3

	Alabama	Wisconsin	Total
Number of Pupils	2630	2140	4770
Mean	24.2	24.5	24.3
Standard Deviation	7.66	7.06	7.40

The ordered distribution of 174 class means is given in Table 24. Some class means were very low and in general their distribution indicates that the PIA Midyear Test was quite hard for third graders.

There are no norms for the test. The frequency distribution of test scores is given in Table 25.

Table 24. Distribution of Class Means for PIA Midyear Test, Grade 3

10.0	21.7	24.5	27.3
12.0	21.7	24.6	27.5
14.7	21.9	24.7	27.5
15.2	21.9	24.7	27.5
15.6	22.0	24.8	27.5
15.6	22.2	24.9	27.6
15.7	22.3	24.9	27.7
15.8	22.4	24.9	27.7
15.9	22.4	25.0	27.7
16.2	22.6	25.1	27.9
16.4	22.6	25.1	28.1
17.9	22.6	25.2	28.1
18.2	22.7	25.3	28.3
18.4	22.7	25.4	28.3
18.8	22.8	25.4	28.4
19.3	22.8	25.5	28.5
19.4	22.8	25.5	28.5
19.5	22.9	25.6	28.8
19.6	22.9	25.8	28.9
19.8	22.9	25.8	28.9
19.9	23.0	25.8	28.9
20.0	23.1	25.9	29.3
20.1	23.1	25.9	29.4
20.1	23.3	26.1	29.4
20.3	23.3	26.1	29.5
20.4	23.3	26.2	29.6
20.4	23.4	26.3	29.7
20.4	23.7	26.3	29.9
20.4	23.8	26.3	30.0
20.4	23.8	26.4	30.5
20.7	23.9	26.5	30.6
20.7	23.9	26.6	30.6
20.9	23.9	26.6	30.6
21.0	23.9	26.6	30.7
21.0	24.0	26.6	30.8
21.2	24.0	26.8	30.9
21.3	24.1	26.8	31.0
21.3	24.1	26.9	31.6
21.3	24.1	26.9	32.1
21.4	24.2	27.0	32.1
21.4	24.3	27.0	32.3
21.5	24.3	27.2	32.3
21.6	24.4	27.3	33.0
21.6	24.4		

Table 25. Frequency Distribution of Scores on PIA Midyear Test, 45 Items, Grade 3

Score Interval	Frequency		Percent		Percent Below	
	Ala.	Wis.	Ala.	Wis.	Ala.	Wis.
45	0	0	0	0	100	100
43-44	6	5	.2	.2	99.8	99.8
41-42	20	8	.8	.4	99.0	99.4
39-40	48	31	1.8	1.4	97.2	98.0
37-38	71	57	2.7	2.7	94.5	95.3
35-36	119	84	4.2	3.9	90.3	91.4
34	81	53	3.1	2.5	87.2	88.9
33	81	65	3.1	3.0	84.1	85.9
32	77	66	2.9	3.1	81.2	82.8
31	100	82	3.8	3.8	77.4	79.0
30	104	85	4.0	4.0	73.4	75.0
29	95	106	3.6	5.0	69.8	70.0
28	94	90	3.6	4.2	66.2	65.8
27	124	116	4.7	5.4	61.5	60.4
26	122	97	4.6	4.5	56.9	55.9
25	120	113	4.6	5.3	52.3	50.6
24	128	96	4.9	4.5	47.4	46.1
23	110	108	4.2	5.0	43.2	41.1
22	113	115	4.3	5.4	38.9	35.7
21	124	101	4.7	4.7	34.2	31.0
20	89	106	3.4	5.0	30.8	26.0
19	129	97	4.9	4.5	25.9	21.5
18	108	88	4.1	4.1	21.8	17.4
17	111	78	4.2	3.6	17.6	13.8
16	91	78	3.5	3.6	14.1	10.2
14-15	149	106	5.7	5.0	8.4	5.2
12-13	126	48	4.8	2.2	3.6	2.9
10-11	52	37	2.0	1.7	1.6	1.2
8-9	24	15	.9	.7	.7	.4
6-7	7	6	.3	.3	.4	.1
4-5	2	3	.1	.1	.3	.0
2-3	0	0	.0	.0	.3	.0
0-1	5	0	.3	.0	.0	.0

Report 6: Patterns in Arithmetic End-of-Year Test, Grade 3

An end-of-year test was developed especially for the PIA program by Educational Testing Service. Before the PIA program was developed certain concepts deemed important for the third grade were decided upon by the staff at the R & D Center. Like the PIA Midyear Test, the End-of-Year Test is an attempt to measure just how successful the program is in teaching these concepts. Test items emphasize program content after midyear.

The items on the End-of-Year Test covered several important content areas emphasized by the Patterns program. Some of these areas, and the associated problems from the End-of-Year Test designed to measure achievement in them, are discussed below.

1) Numeration received considerable emphasis on the Midyear Test and End-of-Year Test. Problems 1, 2, 7, 9, 12, 19, 22, 27, and 36 tested various aspects of numeration. These aspects include ordering, place value and several general notions that depend upon the children's understanding of terminology such as *greater than* and *between*. Although the problems on numeration were still rather difficult, results indicated that the children had made progress in this area since midyear.

2) The children did quite well on all problems relating to fractions. The ideas of equivalent fractions, pictorial representation of a fraction by shading, and fractional relation-

ships were tested by Problems 3, 4, 10, and 17.

3) The results on Problems 5 and 35 show progress was made toward mastery of the distributive law since midyear.

4) All geometry problems on this test were measurement oriented. Problem 14 requires identification of congruent angles in similar triangles. Problem 15 tests the concept of perimeter of a rectangle 5 feet by 8 feet, but 26 is not one of the choices; the correct choice is (2 x 8) + (2 x 5), again emphasizing the idea that a given number has many names. Problems 20 and 32 are also measurement oriented.

Item difficulties for Alabama and Wisconsin were about the same. The results below indicate that the End-of-Year Test was somewhat easier than the Midyear Test.

The PIA, Grade 3, End-of-Year Test was administered to 162 classes, 82 in Alabama and 80 in Wisconsin near the end of May, 1967. Out of a total of 4565 pupils who took the test, 2411 were from Alabama classes and 2154 were from Wisconsin classes. Table 26 summarizes the results on the 40-item test.

Table 26. PIA End-of-Year Test Summary, Grade 3

	Alabama	Wisconsin	Total
Number of Pupils	2411	2154	4565
Mean	27.4	28.4	27.9
Standard Deviation	8.19	7.43	7.86

The ordered distribution of 162 class means is given in Table 27. Some class means were quite low and others impressively high. That the class means range from a low of 12.9 to a high of 37.4 is not particularly surprising because of the design of the study. This list of class means includes both low and high socioeconomic classes from Alabama and Wisconsin.

There are no norms for the test. The frequency distribution of scores for Alabama and Wisconsin is given in Table 28. Note that for Alabama the mode, the most frequently made score, is 31, while the mode for Wisconsin is 29.

Table 27. Distribution of Class Means for PIA End-of-Year Test, Grade 3

12.9	26.0	30.2
13.0	26.1	30.2
14.9	26.2	30.2
15.6	26.4	30.3
16.1	26.7	30.3
16.4	26.7	30.4
17.0	26.9	30.4
17.1	26.9	30.4
17.6	27.1	30.4
19.6	27.1	30.6
19.6	27.1	30.7
19.8	27.1	30.8
20.1	27.2	31.0
20.3	27.3	31.1
20.6	27.3	31.5
20.6	27.4	31.5
21.4	27.4	31.6
21.7	27.5	31.8
21.8	27.5	32.2
22.3	27.9	32.3
22.6	27.9	32.4
22.7	27.9	32.6
22.8	28.1	32.6
22.9	28.1	32.7
23.0	28.1	32.7
23.0	28.1	32.8
23.1	28.1	32.9
23.4	28.2	33.0
23.6	28.5	33.0
23.7	28.6	33.0
23.8	28.6	33.1
23.9	28.6	33.2
24.2	28.7	33.4
24.3	28.8	33.5
24.7	28.8	33.5
24.7	28.9	33.7
24.8	28.9	33.8
24.8	28.9	33.9
24.8	28.9	33.9
24.9	29.0	34.0
25.0	29.0	34.0
25.2	29.0	34.1
25.3	29.0	34.1
25.3	29.3	34.2
25.3	29.3	34.3
25.5	29.6	34.4
25.6	29.6	34.9
25.6	29.6	35.0
25.6	29.8	35.8
25.7	29.8	36.1
25.7	29.9	36.4
25.9	30.0	36.8
26.0	30.1	37.1
26.0	30.1	37.4

Table 28. Frequency Distribution of Scores on PIA, End-of-Year Test, 45 Items, Grade 3

Score Interval	Frequency		Percent		Percent Below	
	Ala.	Wis.	Ala.	Wis.	Ala.	Wis.
44-45	10	4	.4	.2	99.6	99.8
42-43	42	46	1.7	2.1	97.9	97.7
40-41	103	80	4.3	3.7	93.6	94.0
38-39	116	135	4.8	6.3	88.8	87.7
37	86	77	3.6	3.5	85.2	84.2
36	90	64	3.7	2.9	81.5	81.3
35	95	88	3.9	4.1	77.6	77.2
34	102	87	4.2	4.0	73.4	73.2
33	105	101	4.4	4.7	69.0	68.5
32	96	106	4.0	4.9	65.0	63.6
31	108	103	4.5	4.8	60.5	58.8
30	85	95	3.5	4.4	57.0	54.4
29	97	121	4.0	5.6	53.0	48.8
28	105	110	4.4	5.1	48.6	43.7
27	94	100	3.9	4.6	44.7	39.1
26	94	84	3.9	3.9	40.8	35.2
25	86	80	3.6	3.7	37.2	31.5
24	73	80	3.0	3.7	34.1	27.8
23	95	90	3.9	4.2	30.2	23.6
22	94	99	3.9	4.6	26.3	19.0
21	85	62	3.5	2.9	22.8	16.1
20	68	56	2.8	2.6	20.0	13.5
19	74	58	3.1	2.7	16.9	10.8
18	79	41	3.3	1.9	13.6	8.9
17	67	49	2.8	2.3	10.8	6.6
16	66	38	2.7	1.8	8.1	4.8
14-15	94	54	3.9	2.5	4.2	2.3
12-13	57	25	2.3	1.2	1.9	1.1
10-11	26	11	1.1	.5	.8	.6
8-9	8	8	.3	.4	.5	.2
6-7	1	1	.1	.1	.4	.1
4-5	6	1	.2	.1	.2	.0
2-3	2	0	.1	.0	.1	.0
0-1	2	0	.1	.0	.0	.0

Report 7: The Stanford Achievement Tests, Forms W and Y

Pre- and posttests in computation were given to approximately half of the classes participating in the Patterns in Arithmetic field study in the third grade. Forms W and Y of *Test 7* from the *Stanford Achievement Test: Primary II Battery* were used for the pre- and posttest respectively. The two forms of the test are comparable so a measure of growth can be made. Each form of the test contained 60 computation items.

Results of the Testing: Pretest, Form W. The Stanford Achievement, Form W, computation test was administered to 87 classes, 37 in Wisconsin and 50 in Alabama. Out of a total of 2563 pupils who took the test, 1084 were from Wisconsin classes and 1479 were from Alabama classes. Table 29 summarizes the results of the 60-item pretest given near the beginning of the 1966-67 school year.

Table 29. Stanford Achievement Test, Form W, Summary

	Alabama	Wisconsin	Total
Number of Pupils	1479	1084	2563
Mean	19.8	20.9	20.4
Standard Deviation	7.02	7.99	7.62

Table 30 gives the ordered distribution of 87 class means. The frequency distribution of test scores for Alabama and Wisconsin is given in Table 31. Norms for the test are given in Table 32.

It is apparent when the scores from Table 31 are compared with the norms that many children had not mastered elementary computational skills at the beginning of the third grade. The 50th percentile score is 27. In Alabama 75.7% of the children scored below 27 and in Wisconsin 83% scored below 27. The Patterns field study group took the Stanford test soon after school opened in the fall. This meant that very little if any review was possible before the computation test was taken. It is very likely that being away from arithmetic over the summer with no chance for review depressed the scores. Moreover, the norms for the test cover the period from September to December. It is very likely that the norms population took the test several weeks after school opened in the fall. The extra time for review and introduction of new material would naturally make the norms scores higher than they would have been had the test been given very early in the school year as it was for the Patterns children.

Table 30. Distribution of Class Means: Stanford, Form W, Pretest

8.3	18.6	21.3	23.4
8.6	18.6	21.4	23.4
10.1	19.0	21.4	23.7
12.0	19.1	21.5	23.8
12.4	19.3	21.6	24.0
12.8	19.4	21.7	24.1
14.1	19.5	21.8	24.3
14.1	19.6	21.8	24.6
14.9	20.0	21.9	25.0
14.9	20.4	21.9	25.0
15.1	20.4	22.0	25.4
15.1	20.5	22.0	25.4
15.3	20.5	22.1	25.5
16.3	20.6	22.1	25.6
16.7	20.7	22.3	25.7
17.7	20.8	22.6	25.8
17.8	20.8	22.6	26.3
17.8	21.0	22.6	26.4
17.9	21.1	22.7	26.7
18.1	21.2	23.1	26.9
18.3	21.2	23.2	27.6
18.3	21.3	23.3	

Table 31. Frequency Distribution of Scores: Stanford, Form W, Pretest

Score Interval	Frequency		Percent		Percent Below	
	Ala.	Wis.	Ala.	Wis.	Ala.	Wis.
47-60	0	0	0.0	0.0	100	100
45-46	3	0	.2	0.0	99.8	100
43-44	0	0	0.0	0.0	99.8	100
41-42	3	0	.2	0.0	99.6	100
39-40	4	2	.3	.2	99.2	99.8
37-38	3	4	.2	.4	99.0	99.4
35-36	17	8	1.2	.8	97.8	98.6
33-34	32	11	2.1	1.0	95.7	97.6
31-32	73	43	4.9	4.0	90.8	93.6
29-30	115	52	7.8	4.8	83.0	89.1
27-28	108	66	7.3	6.1	75.7	83.0
25-26	142	101	9.6	9.3	61.1	73.7
23-24	185	110	12.5	10.2	53.6	63.5
21-22	164	126	11.0	11.7	42.6	51.8
19-20	142	98	9.6	9.0	33.0	42.8
17-18	113	109	7.6	10.1	25.4	32.7
15-16	87	90	5.9	8.3	19.5	24.4
13-14	73	102	4.9	9.5	14.6	14.9
11-12	51	57	3.5	5.2	11.1	9.7
9-10	45	46	3.1	4.2	8.0	5.5
7-8	21	28	1.4	2.5	6.6	3.0
5-6	25	15	1.7	1.4	4.9	1.6
3-4	25	5	1.6	.5	3.3	1.1
1-2	28	6	1.9	.6	1.4	.5
0	20	5	1.4	.5	.0	.0

Table 32. Fall Norms: Stanford, Form W, Test

Raw Score	Percentile Rank
45	97
41	94
39	89
35	82
31	70
29	60
27	50
25	44
23	38
19	24
16	14
12	6
8	2
4	1

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Results of the Testing: Posttest, Form Y. The Stanford Achievement, Form Y, computation test was administered to 71 classes in Wisconsin and Alabama (34 Wisconsin classes). Out of a total of 2055 pupils who took the test 916 were from Wisconsin classes and 1139 were from Alabama classes. Table 33 summarizes the results of the 60-item posttest, given near the end of the 1966-67 school year.

Table 33. Stanford Achievement Test, Form Y, Summary

	Alabama	Wisconsin	Total
Number of Pupils	1139	916	2055
Mean	41.3	37.6	39.6
Standard Deviation	12.56	9.85	11.58

The ordered distribution of 71 class means is given in Table 34. Table 35 gives the frequency distribution of test scores for Alabama and Wisconsin. Table 36 gives spring norms for the Stanford test.

For several classes, particularly in Alabama, the standard deviation was quite high. This indicates that several teachers were faced with a wide range of pupil abilities. The class range of scores for some Alabama and Wisconsin classes was over 50, and in general these classes had high standard deviations as would be expected. Inspection of Table 35 reveals that test scores are distributed quite differently in Alabama than Wisconsin. In Alabama a greater percentage of the children have high scores than in Wisconsin. For example, in Alabama approximately 40% of the children scored above 46 on the 60-item test, while in Wisconsin only 18% scored above 46.

An interesting comparison can be made from the pre- and posttest norms. Recall that 75.5% of the children in Alabama and 83% of the children in Wisconsin scored below the 50th percentile of 27 on the pretest. On the posttest, using Table 36 for Spring norms, one finds that 34.9% of the children in Alabama and 53% of the children in Wisconsin scored below the 50th percentile score of 40. Figure 3 makes another comparison when Alabama and Wisconsin scores are combined. Several classes, most of which were poor, dropped out of the study after the pretest. This tends to inflate the growth comparison made in Figure 3. From Table 29, 2563 took the pretest, while from Table 33, 2055 took the posttest.

Table 34. Distribution of Class Means: Stanford, Form Y, Posttest

15.4	37.9	43.1
16.2	38.6	43.1
20.9	39.1	43.5
26.6	39.4	43.8
30.6	39.7	44.0
31.4	39.7	44.3
32.0	39.8	44.4
32.2	39.9	44.5
33.8	40.0	44.7
34.3	40.0	44.9
34.3	40.1	45.5
34.5	40.1	45.7
34.6	40.3	46.0
35.0	40.4	47.2
35.3	40.5	47.4
35.3	40.6	47.7
35.4	41.1	48.1
35.6	41.2	49.0
36.1	41.3	49.4
36.5	41.3	49.5
37.1	41.5	49.6
37.2	41.6	52.4
37.2	41.7	53.8
37.3	42.3	

Table 35. Frequency Distribution of Scores: Stanford, Form Y, Posttest

Score Interval	Frequency		Percent		Percent Below	
	Ala.	Wis.	Ala.	Wis.	Ala.	Wis.
59-60	23	1	2.0	.1	98.0	99.9
57-58	50	7	4.4	.7	94.0	99.2
55-56	50	9	4.4	1.0	89.6	98.2
53-54	71	14	6.3	1.6	83.3	96.6
51-52	86	22	7.6	2.4	75.7	94.0
49-50	85	45	7.5	4.9	68.2	89.1
47-48	92	64	8.1	7.0	60.1	82.1
45-46	92	83	8.1	9.0	52.0	73.1
43-44	94	76	8.2	8.3	43.8	64.8
41-42	74	72	6.5	7.8	37.3	57.0
39-40	56	74	4.9	8.1	32.4	48.9
37-38	53	79	4.7	8.7	27.7	40.2
35-36	45	64	3.9	7.0	23.8	33.2
33-34	46	68	4.0	7.4	19.8	25.8
31-32	30	47	2.6	5.2	17.2	20.6
29-30	29	41	2.5	4.5	14.7	16.1
27-28	19	33	1.7	3.6	13.0	12.5
25-26	14	17	1.3	1.9	11.7	10.6
23-24	19	26	1.7	2.8	10.0	7.8
21-22	17	17	1.5	1.9	8.5	5.9
19-20	13	9	1.2	1.0	7.3	4.9
17-18	10	15	.9	1.6	6.4	3.3
15-16	13	10	1.1	1.0	5.3	2.3
13-14	10	8	.9	.8	4.4	1.5
11-12	13	5	1.1	.5	3.3	1.0
9-10	6	3	.6	.3	2.7	.7
7-8	6	4	.6	.4	2.1	.3
5-6	8	0	.7	.0	1.4	.3
0-4	15	3	1.4	.3	.0	.0

Table 36. Spring Norms: Stanford, Form Y, Test

Raw Score	Percentile Rank	Raw Score	Percentile Rank
60	99	36	40
58	99	34	32
56	98	32	28
54	96	30	20
52	94	28	14
50	89	26	11
48	86	24	10
46	77	22	6
44	72	20	4
42	64	18	3
40	50	16	2
38	44	10	1

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	NUMBER PUPILS	MEAN	STDEV	50%ILE SCORE
Pretest ---	2563	20.4	7.62	27
Posttest ▒▒	2055	39.6	11.58	40

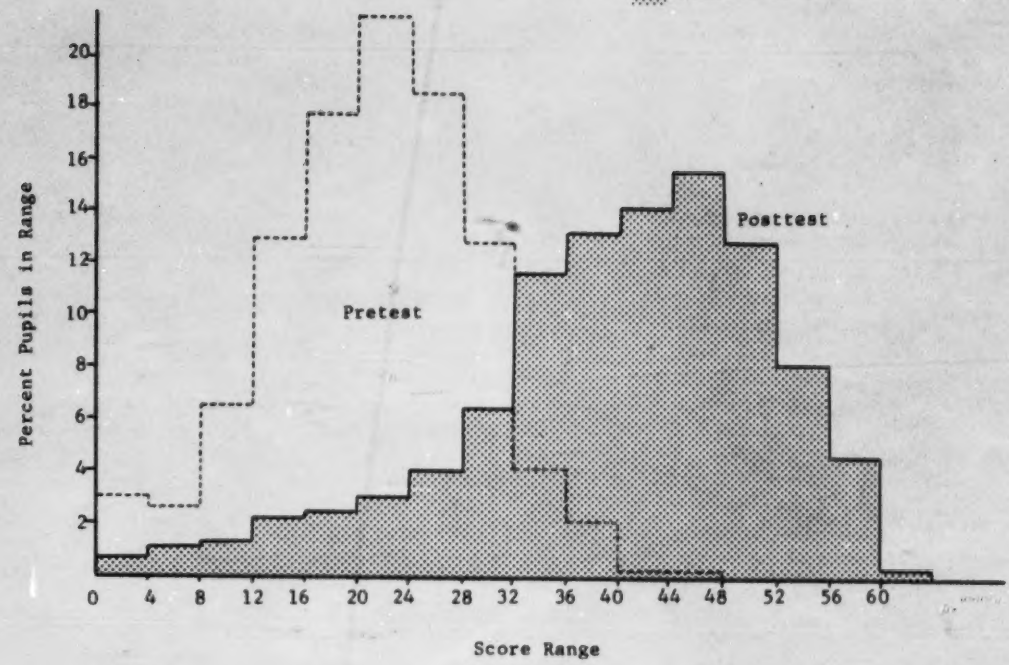

Figure 3. Growth Graph: Stanford Achievement Computation Tests

Frequency distribution of scores on the Stanford, Form W, pretest and Stanford, Form Y, posttest. The interval 28-32 includes those pupils who scored 28, 29, 30, or 31.

Report 8: The ETS Cooperative Primary Tests, Forms 23B and 23A

Pre- and posttests on concepts were given to approximately half of the classes participating in the Patterns in Arithmetic field study in the third grade. Lower Primary, Form 23B was used for the pretest, while Form 23A was used for the posttest. The two forms of the test are comparable so a measure of growth can be made. Each form of the test contained 60 items, most of which tested the pupil's understanding of ideas deemed important at this stage of the child's mathematical development. Of all the standardized tests considered for evaluating the Patterns program, the ETS tests were the most consistent with the objectives of PIA. Sample problems from these tests may be found in Appendix A.

Results of the Testing: Pretest, Form 23B. The ETS Cooperative Primary Test, Form 23B, was administered to 91 third-grade classes in Alabama and Wisconsin (42 Wisconsin classes). Out of a total of 2577 children who took this test, 1143 were from Wisconsin classes and 1434 were from Alabama classes. Table 37 summarizes the results on the 60-item pretest given near the beginning of the 1966-67 school year.

Table 37. ETS Test, Form 23B, Summary

	Alabama	Wisconsin	Total
Number of Pupils	1434	1143	2577
Mean	33.7	34.9	34.2
Standard Deviation	8.33	7.59	8.03

The ordered distribution of 91 class means is given in Table 38. Table 39 gives the frequency distribution of test scores for Alabama and Wisconsin. Although the group means for Alabama and Wisconsin are close, Table 39 shows that the scores are distributed differently; there are more extreme scores in Alabama than in Wisconsin. For example, 24.1% of the children in Alabama scored below 28 while in Wisconsin 15.8% scored below 28.

Fall norms for the test are given in Table 40. From Table 37 the mean for Alabama and Wisconsin combined is 34.2. This is somewhat better than the norms mean of 32.8. The standard deviation for the field study group is 8.03 compared with 8.2 for the norms group. In summary, the field study group did better and was less variable than the norms group at the beginning of the third grade. At the end of the third grade a parallel form of this test was administered to the same children. The results of the posttest are discussed below.

Table 38. Distribution of Class Means on ETS, Form 23B, Pretest

20.5	30.6	33.7	35.3	38.0
24.4	30.8	33.8	35.3	38.5
25.2	30.9	33.9	35.8	38.6
25.4	31.5	33.9	36.1	38.7
25.4	31.9	33.9	36.2	38.9
25.9	32.1	34.0	36.2	39.0
26.3	32.1	34.0	36.2	39.4
26.9	32.3	34.3	36.3	40.0
27.5	32.3	34.5	36.5	40.1
27.7	32.4	34.5	36.6	40.4
27.8	32.7	34.5	36.8	40.8
28.7	32.8	34.7	37.0	41.0
29.0	33.0	34.7	37.1	42.5
29.7	33.1	34.8	37.2	42.9
30.2	33.2	34.9	37.4	43.9
30.4	33.3	35.0	37.6	44.1
30.5	33.3	35.1	37.6	44.1
30.6	33.5	35.2	37.8	46.8
30.6				

Table 39. Frequency Distribution of Scores on ETS, Form 23B, Pretest

Score Interval	Frequency		Percent		Percent Below	
	Ala.	Wis.	Ala.	Wis.	Ala.	Wis.
60	0	0	0.0	0.0	100	100
58-59	1	1	.1	.1	99.9	99.9
56-57	3	2	.2	.2	99.7	99.7
54-55	7	8	.5	.7	99.2	99.0
52-53	17	4	1.2	.3	98.0	98.7
50-51	28	14	2.0	1.2	96.0	97.5
48-49	29	27	2.0	2.4	94.0	95.1
46-47	44	46	3.1	4.0	90.9	91.1
44-45	51	62	3.6	5.4	87.3	85.7
42-43	63	67	4.4	5.8	82.9	79.9
40-41	89	91	6.2	8.0	76.7	71.9
38-39	117	106	8.2	9.2	68.5	62.7
36-37	139	87	9.7	7.6	58.8	55.1
34-35	127	105	8.9	9.2	49.9	45.9
32-33	141	113	9.8	9.9	40.1	36.0
30-31	122	119	8.5	10.4	31.6	25.6
28-29	107	107	7.5	9.4	24.1	15.8
26-27	98	56	6.8	4.9	17.3	10.9
24-25	91	64	6.3	5.6	11.0	5.4
22-23	75	34	5.2	3.0	5.8	2.4
20-21	29	16	2.0	1.4	3.8	1.1
18-19	37	6	2.6	.5	1.2	.6
16-17	5	5	.3	.4	.9	.3
14-15	5	2	.3	.2	.6	.1
12-13	6	1	.4	.1	.2	.0
10-11	1	0	.1	.0	.1	.0
8-9	0	0	.0	.0	.1	.0
6-7	2	0	.1	.0	.0	.0
0-5	0	0	.0	.0	.0	.0

Table 40. ETS National Norms: Grade 3

Fall Pretest		Spring Posttest	
Raw Score Form 23B	Mid-Percentile Rank	Raw Score Form 23A	Mid-Percentile Rank
59-60		59-60	
58		58	99.6
56-57		56-57	99
54-55	99.7	54-55	97
53	99.2	53	94
51-52	99	51-52	92
49-50	98	49-50	88
48	97	48	84
46-47	95	46-47	76
44-45	92	44-45	69
43	90	43	64
41-42	85	41-42	56
39-40	78	40	48
38	74	38-39	45
36-37	67	36-37	37
35	59	35	31
33-34	54	33-34	27
31-32	45	31-32	21
30	38	30	16
28-29	31	28-29	12
26-27	23	26-27	8
25	18	25	6
23-24	13	23-24	5
21-22	8	22	2
20	5	20-21	2
18-19	3	18-19	0.6
16-17	1	17	0.2
15	0.7	15-16	0.1
13-14	0.4	13-14	0.1
11-12	0.3	12	0.1
10	0.3	10-11	0.1
8-9	0.3	8-9	0.1
7	0.2	7	0.1
5-6	0.2	5-6	0.1
3-4	0.2	4	0.1
2	0.1	2-3	0.1
0-1	0.1	0-1	0.1
32.8 Mean		39.5 Mean	
8.2 Standard Deviation		8.9 Standard Deviation	
38.5 Upper Quartile		46.3 Upper Quartile	
32.5 Median		40.2 Median	
27.0 Lower Quartile		32.9 Lower Quartile	

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Results of the Testing: Posttest, Form 23A. The ETS Cooperative Primary Test, Form 23A was administered to 83 third-grade classes in Alabama and Wisconsin (41 Wisconsin classes). Out of a total of 2346 children who took the test, 1215 were from Alabama classes and 1131 were from Wisconsin classes. Table 41 summarizes the results on the 50-item posttest given near the end of the 1966-67 school year.

Table 41. ETS Test, Form 23A, Summary

	Alabama	Wisconsin	Total
Number of Pupils	1215	1131	2346
Mean	43.2	43.8	43.5
Standard Deviation	9.04	7.65	8.40

The ordered distribution of 83 class means is given in Table 42. Table 43 gives the frequency distribution of test scores for Alabama and Wisconsin. There is very little difference in the distribution of scores for Alabama and Wisconsin. From Table 41, the group mean for Alabama is 43.2 and for Wisconsin it is 43.8

Spring norms for the test are given in Table 40. From Table 41, the mean for Alabama and Wisconsin combined is 43.5. This is much better than the norms group mean of 39.5. The standard deviation of 8.4 for the field study group indicates they were slightly less variable than the norms group, which had a standard deviation of 8.9.

Table 42. Distribution of Class Means on ETS, Form 23A Posttest

24.7	40.2	43.2	45.5	47.3
29.2	40.6	43.5	45.8	47.8
29.4	40.6	43.6	45.8	48.0
32.8	40.6	43.7	45.8	48.2
37.1	40.7	43.7	45.8	48.3
37.1	41.0	43.8	45.9	48.3
37.1	41.1	43.9	46.0	48.5
37.3	41.5	43.9	46.2	48.9
38.2	41.6	44.0	46.4	49.2
38.4	41.9	44.0	46.5	49.3
39.3	42.2	44.2	46.6	49.8
39.4	42.4	44.3	46.6	49.9
39.9	42.6	44.4	46.6	50.1
40.0	42.7	44.5	46.8	50.6
40.1	42.7	44.6	47.2	50.8
40.1	42.9	45.3	47.3	51.8
40.1	42.9	45.5		

Table 43. Frequency Distribution of Scores on ETS, Form 23A, Posttest

Score Interval	Frequency		Percent		Percent Below	
	Ala.	Wis.	Ala.	Wis.	Ala.	Wis.
60	1	2	.1	.2	99.9	99.8
58-59	9	6	.7	.5	99.2	99.3
56-57	40	29	2.3	2.6	96.8	96.7
54-55	76	52	6.3	4.6	90.5	92.1
52-53	101	103	8.3	9.1	82.2	83.0
50-51	117	107	9.6	9.5	72.6	73.5
48-49	112	102	9.2	9.0	63.3	64.5
46-47	123	119	10.1	10.5	53.2	54.0
44-45	100	112	8.2	9.9	45.0	44.1
42-43	89	91	7.3	8.0	37.6	36.1
40-41	74	91	6.1	8.0	31.5	28.1
38-39	81	80	6.7	7.1	24.7	21.0
36-37	47	68	3.9	6.0	20.8	15.0
34-35	44	55	3.6	4.9	17.2	10.1
32-33	51	34	4.2	3.0	13.0	17.1
30-31	35	33	2.9	2.9	10.0	4.2
28-29	41	19	3.4	1.7	6.5	2.6
26-27	24	17	2.0	1.5	4.5	1.0
24-25	14	3	1.2	.3	3.2	.8
22-23	13	4	1.1	.4	2.0	.4
20-21	7	1	.6	.1	1.4	.3
18-19	8	1	.7	.1	.7	.2
16-17	2	0	.2	0	.5	.2
14-15	3	0	.2	0	.3	.2
12-13	2	0	.2	0	.1	.2
10-11	1	2	.1	.2	.0	.0
0-9	0	0	.0	.0	.0	.0

Comparison: Pretest vs. Posttest. From Tables 37 and 41 the number of pupils taking the pretest and posttest were 2577 and 2346 respectively. Most of the 231 dropouts occurred in Alabama, but for a group of this size 231 dropouts is not disturbing. Most of the dropouts were low socioeconomic Alabama classes. This tended to make the Alabama group mean on the posttest somewhat higher than it would have been otherwise. The average mean growth for Alabama and Wisconsin combined was 43.5 - 34.2 = 9.3 points on the 60-item test. This compares favorably with the average mean growth for the norms group, which was 39.5 - 32.8 = 6.7 points.

From Table 40 the norms median is 32.5 on the pretest and 40.2 on the posttest. For the field study group, the pretest median was 33 for Alabama and 34.5 for Wisconsin. The posttest median for the field study group was 45.6 for Alabama and Wisconsin.

In summary, the above figures and tables show that the field study group was slightly better than the norms group to start with, and by the end of the year the difference was even more pronounced. Figure 4 compares pre- and posttest performance. Alabama and Wisconsin children are considered together.

	NUMBER PUPILS	MEAN	STDEV	50%ILE SCORE
Pretest ---	2577	34.2	8.02	32.5
Posttest ▒▒	2346	43.2	8.40	40.2

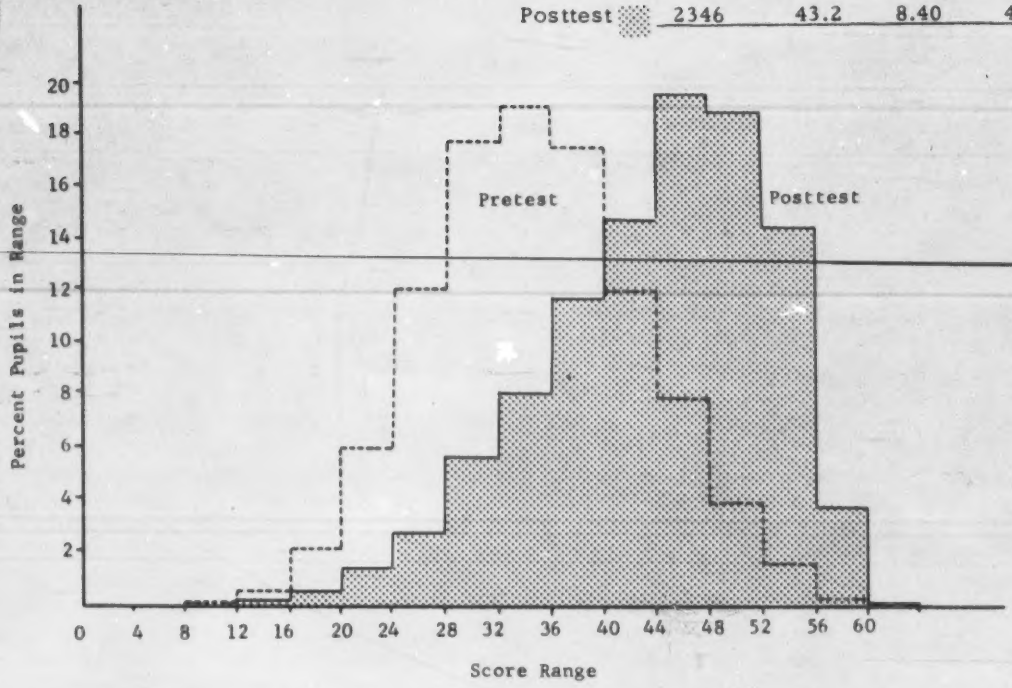

Figure 4. Growth Graph, Educational Testing Service Cooperative Primary Tests, Forms 23B and 23A

Frequency distribution of scores on the ETS Cooperative Primary Tests, Form 23B (Pretest) and Form 23A (Posttest). The interval 28-32 includes those pupils who scored 28, 29, 30, or 31.

VI

MULTIVARIATE ANALYSIS OF DATA

INTRODUCTION

Description of the Analysis

Section III was devoted to a discussion of the results on each test taken in the first- and third-grade studies. No attempt was made in that section to test for differences in test scores between students grouped by state, community size, or socioeconomic status. The purpose of this section is to report such differences or, as the case may be, lack of differences.

The technique used for the statistical evaluation of these differences is multivariate analysis of variance (MANOVA). It is similar to ordinary analysis of variance (ANOVA) in several respects: the observed responses—sometimes called the "dependent" variable—are assumed to be independent, a model depicting the manner in which the "independent" variables affect the average values of the "dependent" variable is assumed to be accurate, and it is assumed that no variables affecting the responses have been ignored. MANOVA differs in that the response is, in this case, not just one test score but three test scores considered jointly. Moreover, the assumption that the responses have equal variances is replaced by the assumption that the vector of response variables has the same covariance matrix from group to group within the population, i.e., that the variances of the three test scores do not vary from group to group (although the variance of scores on one test need not be the same as that for another test) and that the correlations of the three scores are the same in every group. Finally, in order to test hypotheses concerning group differences, the assumption of normality in ANOVA is replaced by the more stringent assumption in MANOVA that each of the response variables not only is normally distributed* but that the joint distribution of the three scores is multivariate normal (even without this assumption, however, the *estimates* of differences—large or small—from group to group are still unbiased and have other desirable statistical properties).

The major purpose of using any statistical technique is to make inferences about a population, e.g., make statements such as "The performance of the students on these tests varies from one state to the other," ***and*** have a good idea of the magnitude of the risk that such a statement is actually wrong, i.e., that it really reflects only, say, the peculiarities of the particular sample of students actually studied. How "good" this notion of magnitude of risk is depends on how well the assumptions of the technique are actually approximated—they are hardly ever realized exactly—in the study.

In this case the most important assumption, that of the independence of the responses of the students, appears to be the one best fulfilled, e.g., the testing was administered to ensure a minimum of "cheating" or copying among students. The weakest aspects of the study are those pertaining to the adequacy of the assumed model, the equality of covariance matrices between groups, and the distribution of the response variable.

The model postulated assumes that responses on the test scores may vary between states, among community size groups, and by socioeconomic status. In studies such as this there is no way of knowing how many other variables affect the responses. It is

*This does not mean that, if one plots any one of the response variables, e.g., the scores from any one of the tests, one would observe a "bell-shaped" distribution. This is because, as postulated by the model, the mean score varies among the various groups, e.g. community sizes, in the sample. It *does* mean that within a particular state and for a particular community size or within a particular socioeconomic group one could expect an approximate normal distribution.

reasonable to assume that there are some other variables which were not measured which affect response and that some of these may be correlated with the independent variables actually included. Hence, it is very important to realize that any differences found cannot be said to be caused by the independent variables. Rather than determining causes, the objective is one of determining whether differences in fact do exist while the problem of explaining these differences will be left for other studies. Also, real differences may be obscured or overemphasized because of the lack of inclusion or control of other variables.

The manner in which socioeconomic status was determined also forms a limitation of this study. This determination was done for all students in an entire school on the basis of information on the Aid to Dependent Children program for the area in which the school was located. Not only is this a very crude and gross method of classification but in communities with only one school such a determination could not be done at all. Thus, the effects of community size and socioeconomic status could not be studied simultaneously, even though the two may be correlated and they may have an interactive affect on the test performances of the students. Finally, because of the method of classification the groups classified as, say, "low socioeconomic status" in Alabama may differ substantially from those so classified in Wisconsin: it is even conceivable that the latter would be more comparable to a middle group in Alabama.

The degree of approximation to the assumed equality of covariance matrices and multivariate normal distribution of the dependent variable is not known. There is some evidence that MANOVA shares the desirable property of ANOVA of being robust to departures from these assumptions. How robust it is probably depends on the kind and magnitude of the departures. Thus, some real differences may be overlooked while some of the differences reported may in fact be very small. Hence, while the conclusions to be reported are the ones which seem most reasonable on the basis of the evidence obtained from the study, they cannot be considered as more than tentative.

Why Use MANOVA?

One might wonder why MANOVA is used here rather than three univariate analyses of variance. The reason is that MANOVA uses the information from the study more efficiently and thus expands our ability to make reliable inferences about the population of students. In particular it utilizes two types of information which the separate analyses of variance do not: (1) the correlations among the scores of the three tests; and (2) what might be termed the cumulative evidence from the three tests taken together.

By using information about the correlations among the three test scores, MANOVA not only "considers" the amount by which a difference departs from zero but also the "direction" of the joint departure. For illustration, suppose there are only two tests and we wish to compare two groups on these tests and that the differences between the groups, expressed in standard scores, are +1.3 on the first and -.95 on the second. When considered separately the ANOVA's would "tell" us that differences of this size or larger would be observed relatively often even when the true differences are zero. However, if the two tests have a moderate *positive* correlation, a MANOVA might reveal that differences in the "direction" observed, i.e., one positive and one negative of the magnitudes given, would occur rarely if the true differences were zero on both tests. On the other hand suppose the values were +1.97 and +1.5. One might conclude on the basis of two separate ANOVA's that the first is "significant" while the second is not, yet the MANOVA might lead us to suspect that such a joint departure in that "direction" when the true differences are both zero is not an unlikely event.

Even if the correlations are small the use of MANOVA has the advantage that the evidence from the three tests used here is considered simultaneously. The observed differences between a high socioeconomic status group and a lower one might take any one of the following forms: (1) The differences on all three tests may be clearly apparent; (2) This may be true for only two tests; (3) This may be true for only one; or (4) The differences are not large on any but they are moderate on all three or on two. The univariate analyses might lead to a rejection of the hypothesis of no differences between the two groups on the three tests in Cases 1, 2, and 3 but not in Case 4. The MANOVA might do so in the fourth case as well, in a sense, for the same reason that a teacher might conclude that pupil A learned more than pupil B in a certain subject only after noting that while A's test grades taken individually are not much greater than B's, the difference in the averages of all the tests is substantial.

If the hypothesis is rejected it may be possible to infer that the groups differ on all three tests, only two, or only one, or that it is not possible to pinpoint the difference in that manner but that the evidence suggests that there is some difference between the groups.

Description of the Output

If the above discussion is clearly in mind, the analyses reported in the following sections and the statistics in Appendix C will be more meaningful. Several computer runs were made to analyze the data. Aspects of these runs that may be of interest can be found in Appendix C. In all, eight computer runs were made: four runs for community size and state analysis, and four runs to determine socioeconomic significance. Each run reported in Appendix C includes the following information:

(1) The factors being investigated (for example, size, state)
(2) The variables (tests taken)
(3) Cell means by factor levels and for each test
(4) Correlation matrix (between tests)
(5) Multivariate F for each hypothesis
(6) Univariate F's for each test
(7) Discriminant function analysis and plot (This aspect will be included only where significance occurs.)

MULTIVARIATE RESULTS: GRADE 1

Recall that classes were divided into two groups, group A and group B (see Figure 2). Group A took the following vector of tests: (PIA Midyear PIA End-of-Year, California Computation). Group B took the vector of tests (PIA Midyear, PIA End-of-Year, ETS 12B). In addition, each class was classified according to community size, state, and socioeconomic status (see Figure 1). The socioeconomic classification was done only in community sizes 3 and 4. The population range for the various community sizes were as follows: Size 1, 0–10,000; Size 2, 10,000–30,000; Size 3, 30,000–90,000; Size 4, 90,000 and above. The results for groups A and B will be discussed separately with respect to size, state, and socioeconomic effects.

Group A

(PIA Midyear, PIA End-of-Year, California Computation) See Run #1 and Run #2, Appendix C.

Hypothesis 1,* Size. There was no significant difference between the mean vectors of test scores across the four sizes. From the cell means in Appendix C, the Wisconsin 10,000 to 30,000 groups did best; the Alabama 10,000 to 30,000 group worst.

Hypothesis 2, State. There was a significant difference between states ($p = .03$) due to better performance by the Wisconsin group on the PIA Midyear Test ($p = .05$) and the California Test ($p = .05$).

Hypothesis 3, Socioeconomic. (See Run #2, Appendix C.) There were significant differences between socioeconomic groups ($p = .05$). There were two sources to account for this: the middle and high socioeconomic groups did significantly better than the low socioeconomic group on the PIA Midyear ($p = .04$) and California Test ($p = .02$) but not the PIA End-of-Year Test. From the cell means in Run #2, Appendix C, it is apparent that the low socioeconomic group did worse than the middle and high socioeconomic groups but that there is little difference between the middle and high socioeconomic groups on the three tests. The 1 degree of freedom tests which actually show that there is no statistically significant difference between the middle and high socioeconomic groups, and that there is a statistically significant difference between the low and high socioeconomic groups, are not included in Appendix C. Only the overall test is included.

From the cell means under Run #2, Appendix C, it follows that the Wisconsin, low socioeconomic, 30,000 to 90,000 group ran contrary to other low socioeconomic groups. This group did best on both the PIA Midyear (25.0) and End-of-Year (27.3) tests. However, there were only two classes in this group.

For this computer run, community sizes 1 and 2 were omitted since these were not classified according to socioeconomic status. It is interesting to observe that the correlation matrices from Run #1 and Run #2 are about the

*The term "hypothesis" is being used informally.

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same. They should be since classes in Run #2 were a subset of those in Run #1. It is surprising that the PIA Midyear Test is more highly correlated with the California Computation Test than with the PIA End-of-Year Test.

Other Aspects. There were no significant interactions among the various factors on Run #1 or Run #2.

Group B

(PIA Mid-Year, PIA End-of-Year, ETS 12B)
See Run #3 and Run #4, Appendix C

Hypothesis 1, Size. There was a difference between the four sizes (significant at the .06 level on the multivariate test and significant at the .07 level on the univariate test performed on PIA Midyear Test scores). The apparent source of this difference was community size 2. This mean for 11 Alabama classes in the 10,000 to 30,000 population range on the PIA Midyear Test was 18.5. The mean for 4 Wisconsin classes in the 10,000 to 30,000 range on this test was 24.1. This makes the group average for community size 2 about 20.0, well below that of the other three community sizes (see cell means, Run #3, Appendix C). In general, Wisconsin classes in the 10,000 to 30,000 population range did much better than corresponding Alabama classes. The reason for this is mainly demographic. In Alabama, communities in the 10,000 to 30,000 range are generally cities apart from large cities. Such cities have the air of a small town. On the other hand, Wisconsin communities in the 10,000 to 30,000 range are generally urban communities and hence quite different from their Alabama counterpart.

Hypothesis 2, State. There was no significant difference between states. For group A there was a significant difference between states. The reasons for this difference are probably the following: (1) groups A and B were different, and (2) there were differences in computation between states in favor of Wisconsin.

Hypothesis 3, Socioeconomic. There was no significant difference between socioeconomic groups on the mean vectors of test scores. There was a definite trend for the high and middle socioeconomic groups to do better than the low socioeconomic group, particularly on the ETS Form 12B Test (See cell means and Univariate F's for Run #4, Appendix C). This result should be encouraging to those who plan to use PIA over a range of abilities.

Other Aspects. There were no significant interactions between the various factors on Run #3 or Run #4.

MULTIVARIATE RESULTS: GRADE 3

Third grade classes in Alabama and Wisconsin were also divided into two groups, Group A and Group B (see Figure 3). Group A took the vector of tests (Stanford, Form W, pretest; PIA Midyear Test; PIA End-of-Year Test; Stanford, Form Y, posttest). Group B took the vector of tests (ETS, Form 23B, pretest; PIA Midyear Test; PIA End-of-Year Test; ETS, Form 23A, posttest).

In the first grade it was not reasonable to use a pretest. To be sure, children know something about arithmetic before they enter first grade, but any attempted measurement of this knowledge would be filled with undesirable error. The main sources of error would be the inability to follow directions and the language barrier. On the other hand, children in the third grade can be pretested with reduced error, i.e. the individual's test score represents more exactly what he knows about the given problems than it would in the first grade.

The main reason for using a pretest is to enter the pretest class mean as a covariate when the data is analyzed. Covariance makes a more reasonable comparison between groups possible by taking their initial ability into account. For example, suppose that at the end of the third grade the means for the low and high socioeconomic groups on the ETS Form 23A test were 38.7 and 42.3 respectively. From these figures one might be able to conclude that the high group did significantly better than the low group—but what does this tell one about the program (treatment)? Suppose, on the other hand, that one knows from a pretest that at the beginning of the year the means of the low socioeconomic group and high socioeconomic groups were 29.3 and 33.1 respectively. Using analysis of covariance one may very likely conclude that at the end of the year there was no significant difference between the socioeconomic groups, i.e. when one takes the initial ability of the two groups into account, the program itself does not create any significant difference between these two groups.

For Group A, the Stanford Form W pretest was used as a covariate. For Group B, the ETS Form 23B pretest was used as a covariate. The technique of multivariate analysis of covariance was used on the remaining

tests given to each group. The results for each group will be discussed separately below.

Group A

(Stanford Form W; PIA Midyear; PIA End-of-Year, Stanford Form Y)

The various hypotheses discussed here consider the last three tests adjusted through multivariate analysis of covariance for the Stanford Form W pretest. Run #5 and Run #6 in Appendix C give cell means for the various groups and supporting data for Group A.

Hypothesis 1, Size. There was no significant difference across the four sizes.

Hypothesis 2, State. There was a highly significant difference between states in favor of Wisconsin ($p = .0001$). The reason for this significance is the significant difference in performance in favor of the Wisconsin group on the PIA Midyear Test ($p = .03$) and the PIA End-of-Year Test ($p = .001$). The Alabama groups went down in their ranking on the PIA tests when compared to their ranking on the Stanford Form W pretest, while the Wisconsin groups went up in ranking. For example, small Alabama communities (size 1) did best (mean = 21.9) on the Stanford pretest, but worst and next to worst on the PIA Midyear and End-of-Year Tests respectively. On the other hand, the Wisconsin classes in size 3 ranked sixth (worst is eighth) on the Stanford pretest, but improved to third on the PIA Midyear Test and second on the PIA End-of-Year Test (see Cell Means, Run #5, Appendix C). Also, from the cell means in Appendix C, it is clear that the Alabama group did better than the Wisconsin group on the Stanford Form Y posttest. However, when the technique of analysis of covariance is used, and class means on the Stanford Form W pretest are entered as covariates, there is no significant difference in performance between Alabama and Wisconsin on computation.

In order to explain the significant between-state difference on the two PIA tests one needs to examine in more detail two important aspects. First, by its very nature, analysis of covariance attempts to modify a posttreatment measure by taking into account a pretreatment measure. The logical question to ask then is, How good is a premeasure on computation for covarying on a concepts test? The results for Group B, Grade 3, provide a partial answer to this question and any additional discussion will be deferred. A second consideration not completely unrelated to the first is teacher emphasis within the classroom. From all appearances Alabama teachers were, in general, more computation oriented. The teacher followup to a large extent determines the success of the TV lesson. If the teacher does not emphasize concepts in the classroom, test results will reflect the lack of emphasis. The fact that many Alabama classes that did well on the computation tests did less well on the concepts test tends to reflect teacher emphasis.

Hypothesis 3, Socioeconomic. Using the Stanford Form W pretest as a covariate there was no significant difference across the three socioeconomic groups. Examination of the cell means table for Run #6 shows that the three socioeconomic groups did much the same on the PIA Midyear, PIA End-of-Year, and Stanford Form Y Tests as they did on the pretest.

Other Aspects. There were no significant interactions among the various factors on Run #5 or Run #6.

Group B

(ETS Form 23B; PIA Midyear; PIA End-of-Year; ETS Form 23A)

The various hypotheses discussed here consider the last three tests adjusted through multivariate analysis of covariance for the ETS Form 23 B pretest. Run #7 and Run #8 in Appendix C give the cell means for the various groups and supporting data for Group B.

Hypothesis 1, Size. Using the ETS Form 23B test as a covariate, there was no significant difference across the four sizes.

Hypothesis 2, State. The multivariate program showed that when the ability of the pupils at the beginning of Grade 3 is considered, the PIA program does not create any significant difference between states. This is reflected by the fact that those groups who scored low on the pretest continued to score low on the remaining tests and those who scored high on the pretest continued to score high.

When the Stanford Form W test was used as a covariate for the Group A analysis it was found that there was a significant difference between states. Significantly better performance

by the Wisconsin group on both the PIA Mid-year and PIA End-of-Year tests accounted for the difference. In the multivariate discussion for Grade 3, Group A, it was pointed out that one explanation for the significant between state difference was the possible inability of a premeasure on computation to adequately predict success on a concepts test. The fact that when the ETS Form 23B concepts test was used as a covariate there was no significant between state difference gives support to this argument.

Hypothesis 3, Socioeconomic. Using covariance there was an overall significant difference between the vectors of test scores for the three socioeconomic groups ($p = .006$). The particular test responsible for this significance was the ETS Form 23A test and not the two PIA tests. The table of univariate F's in Appendix C, Run #8, shows that there was a significant difference between the three socioeconomic groups on the ETS Form 23B test ($p = .002$). In order to see which socioeconomic groups are favored it is necessary to look at the cell means under Run #8 in Appendix C. In particular, the next to last column gives the estimated cell means on the ETS Form 23A test. These are the cell means adjusted for the covariate pretest. The cell means for the high socioeconomic groups are generally much higher than those for either the low or middle socioeconomic groups. In fact, the weighted (by number of classes) estimated means for the low, middle, and high socioeconomic groups are 25.0, 25.6, and 27.7 respectively. Thus, it appears that PIA operated better in high socioeconomic classes than low or middle—at least when a concepts test was used as a covariate.

Other Aspects. The test for interactions was significant ($p = .05$). Apparently the cause of significance was interaction between socioeconomic status and size on the ETS Form 23A test. Within the middle socioeconomic group community size 4 did much better than community size 3, while within the high socioeconomic group community size 3 did better than community size 4 (see cell means, Appendix C, Run #8). There were no significant interactions between size and state on Run #7.

VII

TEACHER AND PUPIL INVENTORIES

Three inventories were administered to participants in the first- and third-grade study. Two of the inventories were responded to by the teachers—the Teacher Information Sheet and Teacher Opinion Inventory—and the other by the children—the Pupil Inventory. The Teacher Information Sheet provided information about the teachers educational background, years of experience, age, in-service work, etc. The purpose of this inventory was to provide information which could be entered as independent variables in the multivariate analysis. Numerical quantities were assigned to the various questions on the Teacher Information Sheet and entered as covariates in the multivariate analysis. Upon looking at the correlation matrices, it was found that none of the teacher covariates was very highly correlated with class performance on the various tests taken. In fact, the correlation of 0.27 between math education courses taken by the teachers and the results of the teacher's class on the ETS Form 12B test was the highest correlation found. Since all of the correlations were quite low, it was decided to omit the teacher variables from the multivariate analysis. Another factor that made it undesirable to use the teacher variables was the nature of the multivariate computer program. The program required that no data be missing. Several teachers did not answer all questions on the Teacher Information Sheet and about 20% did not return the sheet at all.

THE TEACHER OPINION INVENTORY

A Teacher Opinion Inventory was administered at the end of the school year. A random sample of 50 inventories was taken at each grade level and across the two states. The questions cover a variety of attitudes and opinions about the Patterns in Arithmetic TV program. An abbreviated version of the questions and a response summary is given in Table 44.

THE PUPIL INVENTORY

One section of the Teacher Opinion Inventory asked the teacher her opinion of what the children thought about various aspects of TV arithmetic. The Pupil Inventory gave the children a chance to respond to questions about the TV program and exercise manuals. The teacher read each question to the class and the children circled their responses on an answer sheet. Table 45 summarizes the responses on the Pupil Inventory for each grade.

REMARKS

In general, the inventories showed that both teachers and children liked TV arithmetic. Teachers felt that the concepts covered were appropriate and reasonably paced. The children felt the TV teacher helped them learn arithmetic and they enjoyed working on the worksheets. The first-grade program appeared to be aimed at middle ability children, while the third grade was for middle ability to high ability children.

One major objective of the TV tapes is to provide inservice training for teachers as well as instruction for children. Response to the Teacher Opinion Inventory indicates PIA is an effective inservice instrument.

Table 44. Teacher Opinion Inventory*

Question		Grade 1 (%)	Grade 3 (%)
1. General feeling about PIA.	Liked PIA	60	54
	Liked & Disliked	40	44
	Disliked	0	2
2. Learned new principles while teaching PIA.	Many New Principles	16	40
	Some New Principles	70	54
	Not Any New Principles	14	6
3. Was PIA beneficial to school arithmetic program?	Highly Beneficial	40	54
	Somewhat Beneficial	50	46
	Not Beneficial	8	0
4. Was the program beneficial to pupils?	Highly Beneficial	44	46
	Somewhat Beneficial	54	54
	Not Beneficial	2	0
5. Did teachers learn new methods to present new ideas to pupils?	Many New Methods	28	36
	Some New Methods	66	62
	Not Any New Methods	6	2
6. Effect of TV teacher on learning atmosphere.	Beneficial	78	78
	Little Effect	22	20
	Detrimental	0	0
7. Approximate ability level of group.	High Ability	28	29
	Middle Ability	46	50
	Low Ability	26	21

How Do Teachers Feel PIA Operates Across Abilities?

	ABILITY LEVEL								
	HIGH %			MIDDLE %			LOW %		
	First Grade		Third Grade	First Grade		Third Grade	First Grade		Third Grade
1. General pupil attitude toward TV arithmetic	98	Like	94	98	Like	94	87	Like	67
	2	Dislike	6	2	Dislike	6	13	Dislike	33
2. Did pupils enjoy arithmetic more on TV days?	52	More	34	44	More	33	39	More	18
	41	Same	57	52	Same	60	48	Same	64
	7	Less	9	4	Less	7	13	Less	18
3. Did pupils look forward to and enjoy TV instruction?	57	A Lot	52	55	A Lot	40	33	A Lot	27
	50	Somewhat	48	43	Somewhat	60	52	Somewhat	41
	3	Not At All	-	2	Not At All	-	15	Not At All	32
4. How difficult is PIA for various abilities?	-	Too Hard	2	2	Too Hard	10	66	Too Hard	73
	53	About Right	85	90	About Right	90	34	About Right	27
	47	Too Easy	13	8	Too Easy	-	-	Too Easy	-
5. Could teachers handle individual differences?	79	Yes	81	77	Yes	77	59	Yes	44
	21	No	19	23	No	23	41	No	56

(Continued on next page)

Table 44 (continued)

PIA RATED BY TEACHERS ON A 5-POINT SCALE

SCALE: (1) Excellent (2) Good (3) Average (4) Poor (5) Very Poor

Question	Ability Level and Rating (Averaged Across Teachers)					
	HIGH		MIDDLE		LOW	
	First Grade	Third Grade	First Grade	Third Grade	First Grade	Third Grade
1. Suitability of the Concepts Covered	1.8	1.6	2.0	2.0	2.4	2.5
2. Appropriateness of the Concepts Presented	2.1	1.8	2.2	2.1	2.7	2.5
3. Appropriateness of Pacing of Ideas Within Each Telecast	2.0	1.9	2.3	2.4	2.8	2.9
4. Format of Pupil Exercise Sheets	2.3	1.7	2.4	2.1	3.1	3.0
5. Difficulty of Pupil Exercise Sheets	2.4	1.8	2.5	2.2	3.2	3.3
6. Program Use of Visuals to Assist Concept Development	1.6	1.3	1.7	1.5	2.0	1.6
7. Appropriateness of Visuals for Maintaining Interest	1.7	1.6	1.7	1.6	2.0	1.6
8. Articulation Between TV Program and Pupil Exercises	2.2	1.8	2.5	2.1	3.1	2.7

Miscellaneous Questions (Same Rating Scale as Above)

	Rating	
	First Grade	Third Grade
1. Appropriateness of Printed Commentary Sheets as a Preview of Programs to Come	2.3	1.7
2. Effectiveness of the Printed Commentary Sheets as Instructional Aids for the Concepts Involved	2.6	1.9
3. Effectiveness of the TV Program as an Inservice Device	2.0	1.7

*50 Teacher Opinion Inventories sampled at each grade level.

Table 45. Pupil Inventory: Grades 1 and 3

Question		Response		
		Yes (%)	No (%)	Undetermined (%)
1. Do you like to watch arithmetic on TV?	Grade 1	90	7	3
	Grade 3	88	8	4
2. Would you like to have more arithmetic lessons by TV?	Grade 1	69	26	5
	Grade 3	67	26	7
3. Do you think the lessons on TV should be longer?	Grade 1	46	47	7
	Grade 3	47	46	7
4. Do you like arithmetic better on the days when you have a TV lesson?	Grade 1	71	23	6
	Grade 3	78	18	4
5. Do you feel the TV program helps you learn arithmetic?	Grade 1	96	2	2
	Grade 3	95	2	3
6. Does the TV teacher help you understand arithmetic?	Grade 1	93	5	2
	Grade 3	91	6	3
7. Does the TV teacher make you sleepy?	Grade 1	27	66	7
	Grade 3	21	72	7
8. Do you like to watch the TV teacher?	Grade 1	86	11	3
	Grade 3	86	12	2
9. Do you pay as much attention to the TV teacher as to your own teacher?	Grade 1	77	18	5
	Grade 3	64	29	7
10. Do you like to work on the worksheets?	Grade 1	86	11	3
	Grade 3	80	15	5
11. Do you like the pictures on the worksheets?	Grade 1	88	10	2
	Grade 3	83	13	4
12. Do the worksheets make you tired?	Grade 1	27	66	7
	Grade 3	29	66	5
13. Would you like to work on more worksheets?	Grade 1	68	28	4
	Grade 3	53	42	5
14. Are there too many worksheets for you to do?	Grade 1	27	69	4
	Grade 3	24	71	5
15. Do you think the worksheets are too hard?	Grade 1	20	77	3
	Grade 3	19	78	3

Note.—604 children sampled in Grade 1
635 children sampled in Grade 3

VIII

SUMMARY

During the 1966–67 school year a summative, noncomparative evaluation was done on the Patterns in Arithmetic (PIA) TV program in Grades 1 and 3. Numerous classes in Alabama and Wisconsin participated in the study at both grade levels. The instruments used in the evaluation fall into three categories: (1) achievement tests designed specifically for the PIA program, (2) standardized achievement tests, and (3) teacher and pupil inventories.

In addition to the state variable the PIA staff was interested in how operational the program was in various socioeconomic groups and community sizes. Each class was classified as belonging to one of four community sizes. If a class belonged to one of the two larger community size categories, it was also classified into one of three socioeconomic categories: high, middle or low.

In the first grade the study group compared quite favorably with the norms group on both standardized computation and concepts tests. Not only did the group as a whole compare favorably with norms, but Alabama and Wisconsin classes considered separately compared favorably as well. Tests were constructed by Educational Testing Service for PIA to measure program achievement mainly in the concepts domain. These tests were difficult, but relative to their difficulty the results show that most of the objectives set forth by the PIA staff are being accomplished. The statistical analysis of data in Grade 1 showed no significant differences across the four community size groups. There was a significant difference between states in favor of Wisconsin on the standardized computation test, but not the standardized concepts test. Results tended to favor the middle and high socioeconomic groups over the low socioeconomic group. The differences were more pronounced on the standardized computation and concepts test than on the tests designed specifically for PIA by Educational Testing Service.

In the third grade the study group was also compared favorably with the norms group. Pretesting done in Grade 3 showed both Alabama and Wisconsin students did slightly better than the norms groups on the standardized concepts test, but compared unfavorably with the norms group on the standardized computation test. During the year reasonable growth was made by children in both Alabama and Wisconsin and across the various socioeconomic groups. By the end of Grade 3 the Alabama children were clearly better than the norms group on both computation and concepts tests. The Wisconsin children did clearly better than the norms group on the concepts test, but were slightly below norms on the computation test. This latter result is due to the fact that at the beginning of Grade 3 the Wisconsin group was well below norms on computation. They did show considerable improvement over the year. As for the first grade, the tests designed specifically to measure program achievement were difficult. Item analysis showed the major objectives of PIA are being accomplished.

The statistical analysis of data in Grade 3 used the pretest as a covariate. For some classes the pretest was a concepts test, while for others it was a computation test. In either case there was no significant difference across the four community sizes. There was a highly significant difference between states in favor of Wisconsin when the computation test was used as a covariate. The Wisconsin group did significantly better on tests developed by Educational Testing Service to measure program achievement. When a computation test was used as a covariate there was no significant difference across the three socioeconomic groups. However, when the concepts test was used as a covariate, there was significant difference between socioeconomic groups. It appears that even with the covariate eliminated, the PIA program operated better within the high socioeconomic group than the low or middle.

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In both grades the opinion inventories showed that teachers and children liked TV arithmetic. Teachers felt that the concepts covered were appropriate and reasonably paced. The children felt that the TV teacher helped them learn arithmetic and they enjoyed working on the worksheets. The first-grade program appeared to be aimed at middle ability children, while the third-grade program was for middle to high ability children. One major objective of the TV tapes is to provide inservice training for teachers as well as instruction for children. Response to the Teacher Opinion Inventory indicates PIA is an effective inservice instrument.

APPENDIX A
SAMPLE TEST ITEMS

Sample items from each of the tests used are included in this appendix. Copies of the commercial tests can be obtained from the publisher (see Section III for publisher). Copies of the PIA Tests will be incorporated into a Practical Paper to be available from the R & D Center.

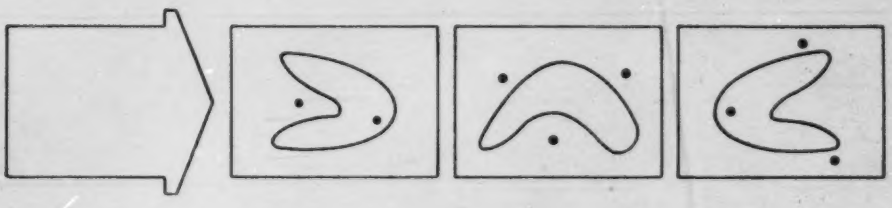
GRADE 1 TESTS

PIA Midyear Test

Each question is read by the teacher and the child responds by marking a large X over his choice.

Item 9: Which shows *three plus four*? . . . *three plus four*.

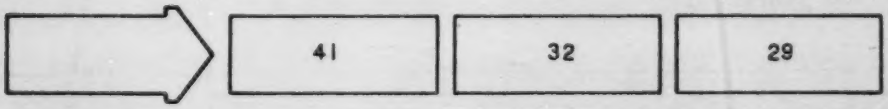
Item 20: Which shows ***only two points*** on the ***same side*** of the curve. Mark the box that shows ***no more than two points*** on the ***same side*** of the curve.



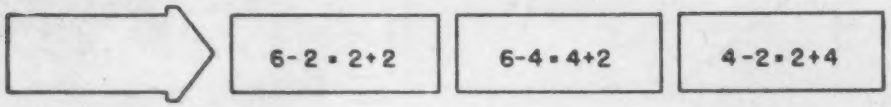
PIA End-of-Year Test

Each question is read by the teacher and the child responds by marking a large X over his choice.

Item 4: Which number is the *greatest*? Mark the box that has the *greatest* of the three numbers.



Item 32: Which box shows a *true* number sentence? . . . a true *number* sentence.



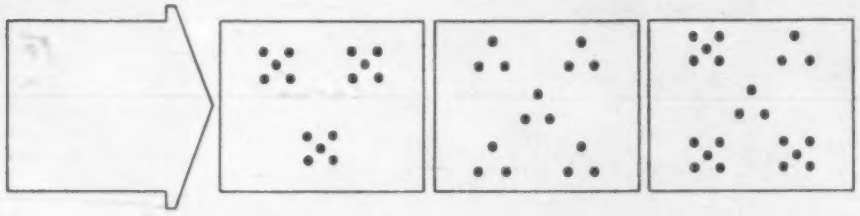
California Achievement Test
Computation Section

Item 7:	Item 21:	Item 38:	Item 44:
6	13	8	47
+2	+ 5	-5	-35

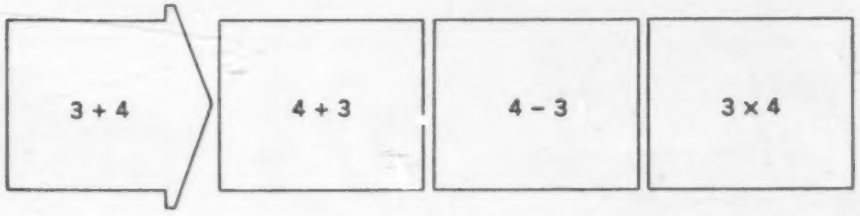
ETS, Cooperative Primary Test, Form 12B

Questions 1–42 are read by the teacher and the child responds by marking a large X over his choice. Questions 43–55 require the child to match the appropriate box with the stimulus in the arrow.

Item 19: Which shows that *fifteen* equals *five* groups of three? . . . that *fifteen* equals five groups of three.



Item 51: (Child looks at stimulus in the arrow and matches it with the appropriate box.)



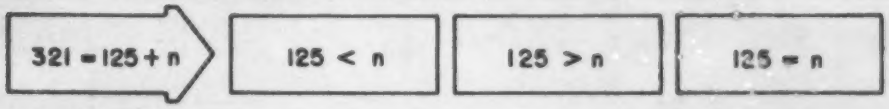
GRADE 3 TESTS

PIA Midyear Test

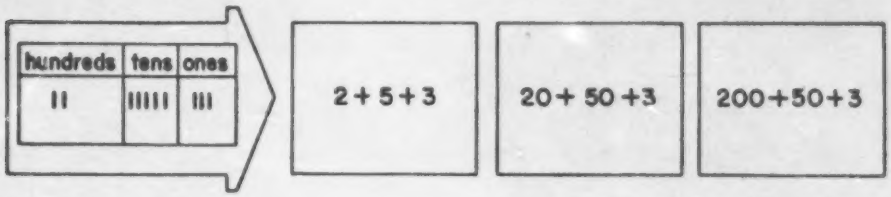
Questions 1–30 are read by the teacher and the child responds by marking a large X over his choice. Questions 31–45 require the child to match the appropriate box with the stimulus in the arrow.

Questions 19 and 51 from Cooperative Primary Tests in Mathematics. Copyright © 1965 by Educational Testing Service, Princeton, N.J. Reprinted with permission of the publisher.

Item 13: If the number sentence in the *arrow* is *true*, which sentence in the *boxes* is *also true*? Which box has a true sentence?



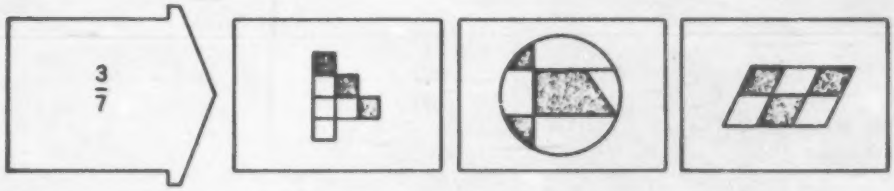
Item 33: (Child looks at stimulus in the arrow and matches it with the appropriate box.)



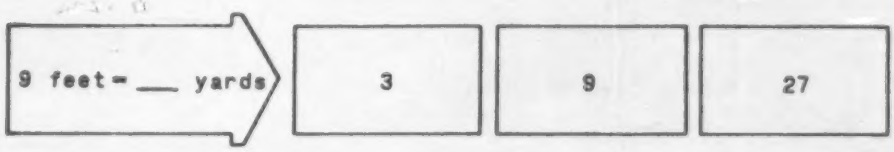
PIA End-of-Year Test

Questions 1–30 are read by the teacher and the child responds by marking a large X over his choice. Questions 31–45 require the child to match the appropriate box with the stimulus in the arrow.

Item 10: In which box is *three-sevenths* of the figure *shaded*? . . . *three-sevenths shaded.*



Item 32: (Child looks at stimulus in the arrow and matches it with the appropriate box.)



Stanford Achievement Tests,
Primary II Battery, Forms W and Y

Forms W and Y are parallel forms of a 60-item computation test. Sample items from each form are included.

Form W Item 9: $\begin{array}{r} 83 \\ +35 \\ \hline \end{array}$ Item 39: $\square \div 2 = 7$

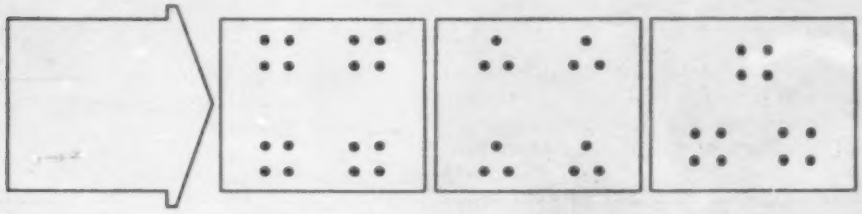
Form Y Item 27: $\begin{array}{r} 119 \\ -62 \\ \hline \end{array}$ Item 55: $148 \div 4 =$ _____

ETS, Cooperative Primary Tests, Forms 23A and 23B

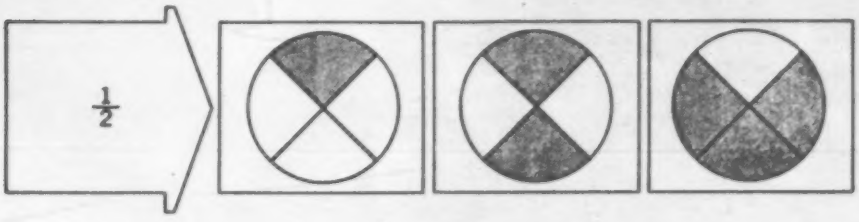
These are parallel forms of a 60-item concepts test. Sample items from each form are included. Questions 1–43 are read by the teachers and the child responds by marking a large X over his choice. Questions 44–60 require the child to match the appropriate box with the stimulus in the arrow.

Form 23A

Item 13: Which is a picture of a number *times itself*? . . . a number *times itself*.

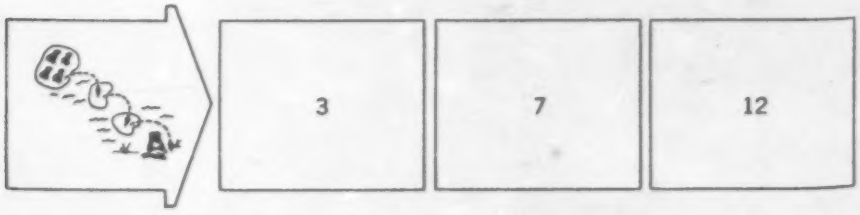


Item 46: (Child looks at stimulus in the arrow and matches it with the appropriate box.)

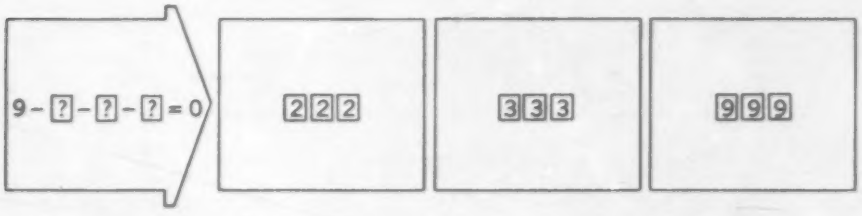


Form 23B

Item 20: Look at the *four* baby frogs on the lily pad. They all want to get to their mother. If they all jump to their mother as shown by the arrows, how many jumps will be made *in all*?



Item 50: (Child looks at stimulus in the arrow and matches it with the appropriate box.)



Items 13 and 46, 20 and 50 from Cooperative Primary Tests in Mathematics. Copyright © 1965 by Educational Testing Service, Princeton, N.J. Reprinted with permission of the publisher.

APPENDIX B

ITEM DIFFICULTY TABLES

This appendix, containing the item difficulties by state for each test given in the first- and third-grade study, will be more meaningful if copies of the various tests are on hand. With a few exceptions, the item difficulties were about the same for Alabama and Wisconsin.

Table 1. Item Difficulties: PIA Midyear Test, Grade 1

Item	Percent Correct	
	Alabama	Wisconsin
1	64.4	75.9
2	67.8	67.0
3	78.6	82.4
4	69.4	68.3
5	43.1	45.7
6	66.3	65.7
7	60.2	69.4
8	47.1	47.3
9	32.5	42.6
10	27.1	25.7
11	88.5	92.9
12	50.0	62.0
13	81.3	88.7
14	85.3	92.2
15	23.9	23.8
16	58.7	61.9
17	44.2	42.7
18	71.5	74.5
19	64.8	69.1
20	42.6	49.1
21	40.7	48.3
22	80.0	84.7
23	34.9	33.7
24	53.4	54.3
25	54.9	65.4
26	53.8	63.3
27	29.9	26.9
28	67.2	72.7
29	37.0	34.5
30	49.3	64.2
31	63.4	61.8
32	27.4	29.3
33	58.6	61.1
34	51.7	49.5
35	75.2	83.9
36	56.7	66.0
37	37.4	44.6
38	24.2	15.5
39	56.6	58.4
40	17.4	12.9

Table 2. Item Difficulties: PIA End-of-Year Test, Grade 1

Item	Percent Correct	
	Alabama	Wisconsin
1	88.2	90.4
2	94.2	94.1
3	75.2	77.3
4	63.4	57.5
5	63.4	71.0
6	82.1	86.3
7	65.8	65.8
8	59.8	58.6
9	52.4	69.6
10	66.3	64.7
11	48.8	47.1
12	56.7	61.8
13	47.3	47.0
14	74.6	76.3
15	53.3	57.2
16	45.8	51.3
17	47.0	57.1
18	75.1	76.6
19	39.7	33.8
20	50.7	52.3
21	86.1	87.6
22	79.0	85.2
23	55.3	61.2
24	47.5	44.2
25	61.8	62.6
26	42.6	38.0
27	33.8	30.5
28	76.5	77.8
29	67.6	71.5
30	55.2	57.0
31	28.9	23.0
32	35.6	30.2
33	55.7	56.1
34	58.0	52.4
35	49.0	48.6
36	50.2	48.9
37	37.3	35.4
38	64.6	64.8
39	45.2	40.9
40	45.1	38.3

Table 3. Item Difficulties: California Test

Item	Percent Correct	
	Alabama	Wisconsin
1	93.5	97.6
2	89.6	96.5
3	90.2	97.3
4	87.7	96.2
5	89.2	96.5
6	82.9	92.2
7	83.1	94.2
8	82.4	91.7
9	85.0	93.1
10	86.4	94.4
11	82.0	91.4
12	81.7	93.2
13	81.7	90.5
14	84.6	94.0
15	82.2	89.4
16	85.3	93.0
17	76.4	84.5
18	68.9	82.6
19	80.0	89.9
20	66.2	75.4
21	55.9	59.5
22	39.1	33.8
23	35.9	42.2
24	34.8	37.5
25	7.5	7.0
26	91.8	95.6
27	77.3	86.4
28	78.0	83.5
29	76.1	85.8
30	77.8	83.3
31	78.9	84.0
32	75.3	88.1
33	69.8	81.8
34	71.5	81.0
35	65.7	76.5
36	73.7	81.9
37	71.6	82.2
38	71.7	79.8
39	69.1	75.7
40	69.0	76.2
41	44.9	43.1
42	28.6	30.7
43	33.6	30.2
44	25.0	26.7
45	3.0	3.1

Table 4. Item Difficulties: ETS, Form 12B, Test

Item	Percent Correct	
	Alabama	Wisconsin
1	95.6	97.3
2	78.7	83.2
3	75.7	77.0
4	80.8	87.6
5	63.5	63.0
6	94.7	97.5
7	94.3	96.3
8	62.1	64.0
9	84.4	87.9
10	88.4	90.2
11	50.5	45.7
12	86.9	91.1
13	62.1	60.5
14	59.8	59.1
15	84.5	82.9
16	75.0	74.9
17	81.6	84.1
18	79.2	82.7
19	64.0	60.6
20	64.7	62.0
21	40.5	35.8
22	55.3	54.2
23	45.0	39.1
24	65.7	68.1
25	72.3	72.0
26	51.5	53.0
27	77.1	72.2
28	72.9	66.9
29	56.2	56.6
30	35.3	27.7
31	71.8	57.4
32	69.4	54.9
33	49.4	50.7
34	63.3	62.2
35	62.6	64.5
36	67.9	76.9
37	52.7	67.4
38	71.5	75.7
39	77.0	85.6
40	89.3	93.9
41	84.0	90.4
42	52.5	43.9
43	85.8	89.0
44	87.6	88.8
45	84.6	88.6
46	53.5	53.7
47	52.9	44.8
48	89.5	93.7
49	91.2	95.0
50	24.8	28.0
51	76.7	79.0
52	61.6	65.1
53	62.0	59.7
54	83.4	86.9
55	79.6	81.4

Table 5. Item Difficulties: PIA Midyear Test, Grade 3

Item	Percent Correct	
	Alabama	Wisconsin
1	91.3	93.4
2	76.8	77.1
3	60.6	55.0
4	47.8	41.8
5	58.4	65.0
6	54.3	50.7
7	88.9	90.7
8	59.5	71.9
9	58.2	61.8
10	59.9	56.6
11	45.9	46.2
12	35.7	36.7
13	33.5	32.1
14	49.2	48.5
15	72.9	70.5
16	30.9	23.1
17	81.0	79.3
18	14.3	10.9
19	46.6	45.2
20	27.7	35.6
21	49.0	50.7
22	53.9	65.2
23	71.0	74.7
24	56.3	57.4
25	46.3	50.5
26	32.2	32.0
27	79.5	80.7
28	41.0	35.9
29	34.6	33.5
30	20.9	19.8
31	87.0	86.9
32	73.3	71.8
33	48.5	50.5
34	56.0	56.6
35	68.8	74.6
36	67.1	63.0
37	64.4	71.0
38	63.2	63.5
39	42.9	40.5
40	36.5	36.4
41	39.2	33.9
42	63.1	66.5
43	41.5	40.1
44	41.7	44.2
45	48.9	56.7

Table 6. Item Difficulties: PIA End-of-Year Test, Grade 3

Item	Percent Correct	
	Alabama	Wisconsin
1	82.1	86.6
2	72.1	76.2
3	74.1	84.1
4	86.6	89.8
5	56.2	61.3
6	77.9	81.3
7	58.0	55.6
8	64.9	65.6
9	60.2	57.3
10	82.4	85.7
11	82.5	85.9
12	55.9	59.5
13	44.8	50.6
14	58.5	58.0
15	59.4	65.5
16	71.4	74.1
17	78.2	83.4
18	45.9	41.3
19	69.3	74.9
20	64.3	56.6
21	44.9	43.7
22	34.4	37.1
23	48.7	52.3
24	32.0	40.3
25	60.7	57.1
26	57.7	54.4
27	41.4	41.7
28	33.3	39.3
29	46.3	47.7
30	34.8	32.1
31	84.9	91.1
32	76.6	82.9
33	78.4	78.2
34	75.7	75.4
35	65.6	69.2
36	51.1	59.4
37	67.6	76.5
38	74.3	80.7
39	67.6	72.6
40	71.0	65.1
41	61.6	67.4
42	36.5	33.7
43	62.5	62.2
44	49.9	49.4
45	36.9	36.7

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Table 7. Item Difficulties: Stanford Achievement Test

FORM W						FORM Y					
	% Correct			% Correct			% Correct			% Correct	
Item	Ala.	Wisc.	Item	Ala.	Wisc.	Item	Ala.	Wisc.	Item	Ala.	Wisc.
1	90.2	89.3	31	29.0	20.4	1	90.6	93.4	31	74.7	72.7
2	85.7	86.0	32	26.3	19.3	2	89.9	92.0	32	77.3	76.6
3	86.8	87.5	33	11.7	12.1	3	91.5	94.1	33	76.3	73.5
4	81.7	84.3	34	5.1	6.0	4	88.8	91.3	34	70.1	68.2
5	79.4	79.3	35	2.3	2.5	5	88.7	91.0	35	61.3	63.6
6	80.7	80.2	36	.9	1.2	6	87.6	90.9	36	50.4	51.0
7	85.0	88.1	37	2.8	3.2	7	96.5	96.7	37	54.4	50.7
8	75.2	78.4	38	13.3	9.9	8	89.8	91.6	38	78.1	75.7
9	71.6	63.8	39	3.0	3.6	9	92.2	92.9	39	48.0	46.3
10	85.1	82.0	40	5.7	4.6	10	91.9	93.1	40	65.1	62.6
11	72.5	65.6	41	2.3	1.9	11	92.0	94.4	41	45.7	39.3
12	65.7	70.4	42	1.1	.6	12	88.0	89.2	42	62.2	45.7
13	70.0	55.4	43	1.8	.1	13	85.8	88.2	43	55.2	22.8
14	63.7	56.6	44	1.1	.2	14	88.4	89.7	44	62.5	45.9
15	72.7	59.0	45	1.1	.1	15	88.0	88.5	45	55.6	21.0
16	76.6	80.4	46	1.0	.3	16	89.6	91.5	46	53.0	36.0
17	70.9	64.5	47	.5	0.0	17	87.8	85.4	47	56.4	23.3
18	70.1	63.4	48	.7	.3	18	88.2	88.0	48	57.3	41.6
19	68.0	60.2	49	.5	0.0	19	89.6	87.2	49	53.1	19.4
20	66.9	56.5	50	.3	.1	20	86.1	86.7	50	49.2	32.8
21	64.4	64.6	51	.7	0.0	21	87.6	89.5	51	54.9	18.7
22	62.4	63.7	52	.5	.5	22	88.2	89.6	52	60.6	43.7
23	61.4	65.2	53	.1	0.0	23	86.7	88.5	53	18.7	6.8
24	61.4	64.3	54	.2	.1	24	84.7	86.7	54	43.7	32.3
25	60.6	62.7	55	.1	0.0	25	85.6	86.0	55	12.8	3.2
26	37.2	38.4	56	.1	0.0	26	75.3	76.2	56	21.2	4.6
27	27.1	22.0	57	.1	0.0	27	76.5	74.1	57	39.9	22.6
28	27.9	20.6	58	.1	0.0	28	76.7	74.9	58	15.8	2.3
29	27.0	20.8	59	.1	0.0	29	74.1	73.0	59	36.7	23.0
30	27.3	20.8	60	.1	0.0	30	78.2	76.6	60	14.7	2.2

Table 8. Item Difficulties: ETS Cooperative Primary Test

Form 23B						Form 23A					
	% Correct			% Correct			% Correct			% Correct	
Item	Ala.	Wisc.	Item	Ala.	Wisc.	Item	Ala.	Wisc.	Item	Ala.	Wisc.
1	88.8	95.7	31	63.2	62.8	1	90.9	98.1	31	84.6	83.8
2	90.9	93.1	32	59.2	61.3	2	92.9	95.0	32	82.6	81.0
3	61.3	55.6	33	41.8	40.4	3	90.3	91.6	33	70.1	72.0
4	71.2	82.9	34	46.7	52.2	4	70.2	67.8	34	83.0	83.9
5	28.4	30.8	35	58.8	54.8	5	80.1	80.7	35	54.7	59.9
6	39.7	42.4	36	63.2	64.0	6	83.7	88.6	36	91.3	92.8
7	75.7	70.8	37	70.6	74.0	7	70.9	63.5	37	61.8	61.8
8	75.0	72.5	38	78.9	88.5	8	75.8	79.0	38	78.9	85.6
9	76.2	82.6	39	93.2	96.8	9	68.6	70.2	39	92.7	95.1
10	61.4	68.8	40	60.0	58.7	10	67.2	73.2	40	83.5	88.8
11	82.5	88.9	41	31.7	39.5	11	85.4	91.9	41	61.1	72.1
12	67.1	66.3	42	42.6	53.2	12	75.7	66.5	42	35.2	36.3
13	81.4	87.6	43	78.8	82.2	13	69.6	76.4	43	57.0	62.9
14	75.2	73.0	44	46.5	46.1	14	78.5	80.4	44	88.5	94.3
15	44.8	40.5	45	22.6	21.1	15	82.1	86.1	45	86.7	90.5
16	60.3	60.9	46	42.3	48.6	16	58.1	49.7	46	63.6	49.6
17	31.5	29.3	47	37.3	45.7	17	75.1	69.1	47	51.5	50.0
18	57.3	58.3	48	50.3	32.4	18	74.0	68.3	48	70.5	81.0
19	39.3	39.5	49	32.1	30.6	19	41.6	39.7	49	76.4	86.5
20	28.7	36.0	50	42.1	46.8	20	93.7	93.5	50	40.7	51.2
21	41.0	43.1	51	35.2	40.0	21	85.7	86.2	51	76.0	55.4
22	92.9	93.3	52	33.5	35.3	22	77.0	71.4	52	71.3	64.0
23	61.6	68.0	53	46.9	50.6	23	72.9	74.6	53	85.1	86.6
24	70.4	68.5	54	63.1	62.1	24	81.4	81.4	54	54.6	53.8
25	51.3	51.5	55	75.0	80.3	25	70.0	66.4	55	44.6	54.6
26	25.0	16.7	56	34.0	35.5	26	65.6	67.2	56	90.4	94.9
27	76.8	80.7	57	72.7	71.2	27	64.4	66.0	57	64.4	64.5
28	51.7	60.0	58	44.6	45.7	28	67.2	69.1	58	64.8	64.6
29	26.7	23.9	59	40.7	42.7	29	71.9	66.1	59	66.1	73.5
30	80.3	87.0	60	47.4	56.3	30	49.9	44.9	60	59.2	66.4

APPENDIX C
MULTIVARIATE DATA*

MANOVA Run #1: Grade 1, Group A (PIA Midyear, PIA End-of-Year, California Computation), Factors: Size (4), State (2)

Cell Means

Size	State	PIAMid	PIAEnd	Calif.	Classes (#)
1	Ala.	22.1	25.4	33.5	7
1	Wis.	22.4	22.9	33.1	9
2	Ala.	20.0	21.5	28.8	13
2	Wis.	24.7	24.8	35.1	5
3	Ala.	21.4	23.4	31.3	9
3	Wis.	23.3	24.2	33.8	12
4	Ala.	20.8	22.9	31.5	13
4	Wis.	21.7	22.6	34.0	16

Correlation Matrix (Between Tests)

	PIAMid	PIAEnd	Calif.
PIAMid	1.0		
PIAEnd	.73	1.0	
Calif.	.78	.72	1.0

Multivariate F

Hypothesis 1: Between sizes 1, 2, 3, and 4.
F-Ratio for multivariate test of equality of mean vectors = 0.9211.
Degrees Freedom: 9 and 180.2
Not significant, $p < 0.51$

Univariate F's

Test	Between Mean Sqs.	Univariate F	p less than
PIAMid	8.3	0.58	0.63
PIAEnd	12.2	0.75	0.53
Calif.	28.5	0.85	0.47

3 Degrees Freedom for Hypothesis, 76 for Error

*Jeremy D. Finn's "Multivariance: Fortran Program for Univariate and Multivariate Analysis of Variance and Covariance" was used for the multivariate analysis. The program came from the School of Education, State University of New York at Buffalo.

Hypothesis 2: Between States (Alabama and Wisconsin)
F-Ratio for multivariate equality of mean vectors = 3.14
Degrees Freedom: 3 and 74
Significant, $p < 0.03$

Univariate F's

Test	Between Mean Sqs.	Univariate F	p less than
PIAMid	61.3	4.29	0.04*
PIAEnd	1.0	0.06	0.80
Calif.	138.5	4.12	0.05*

1 Degree of Freedom for Hypothesis, 74 for Error

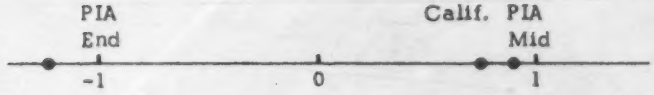
Discriminant Analysis for Hypothesis 2

Canonical Variate 1: Variance = 0.13; Percent of canonical variation = 100

Discriminant Coefficients

Test	Standardized Coefficients
PIAMid	0.90
PIAEnd	-1.22
Calif.	0.75

Discriminant Plot



Other Hypotheses: There were no significant interactions and results are not included.

MANOVA Run #2: Grade 1, Group A (PIA Midyear, PIA End-of-Year, California Computation), Factors: Size (2), State (2), Socioeconomic (3)

Cell Means

Size	State	Socioeconomic	PIAMid	PIAEnd	Calif.	Classes (#)
3	Ala.	Low	19.8	23.5	27.8	5
4	Ala.	Low	17.7	20.2	27.2	4
3	Wis.	Low	25.0	27.3	33.3	2
4	Wis.	Low	18.3	20.3	31.2	3
3	Ala.	Middle	-	-	-	Empty
4	Ala.	Middle	23.5	24.2	32.9	4
3	Wis.	Middle	21.1	22.2	32.9	5
4	Wis.	Middle	23.3	23.1	35.2	9
3	Ala.	High	23.4	23.3	35.6	4
4	Ala.	High	21.1	24.0	33.9	5
3	Wis.	High	24.9	25.0	35.0	5
4	Wis.	High	20.6	23.1	33.4	4

Correlation Matrix (Between Tests)

	PIAMid	PIAEnd	Calif.
PIAMid	1.0		
PIAEnd	0.73	1.0	
Calif.	0.81	0.73	1.0

Multivariate F

Hypothesis 3: Between socioeconomic groups (High, Middle, Low)
F-Ratio for multivariate test of equality of mean vectors = 2.21
Degrees Freedom: 6 and 74
Significant, $p < 0.05$

Univariate F's

Test	Between Mean Sqs.	Univariate F	p less than
PIAMid	45.9	3.4	0.04*
PIAEnd	9.0	0.6	0.58
Calif.	133.6	4.4	0.02*

2 Degrees of Freedom for Hypothesis, 39 for Error

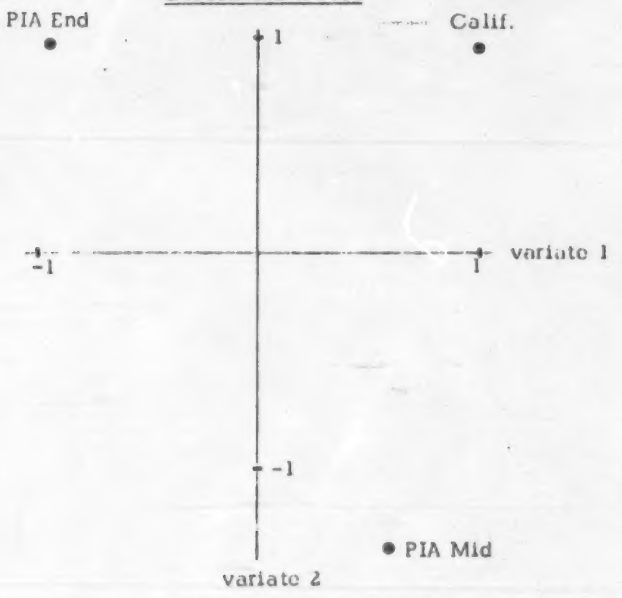
Discriminate Analysis for Hypothesis 3

Canonical Variate 1: Variance = 0.35; Percent of Canonical Variation = 92.6
Canonical Variate 2: Variance = 0.03; Percent of Canonical Variation = 7.4

Discriminant Coefficients
Standardized Coefficients

Test	Variate 1	Variate 2
PIAMid	0.60	-1.33
PIAEnd	-0.95	0.97
Calif.	1.00	0.93

Discriminant Plot



MANOVA Run #3: Grade 1, Group B (PIA Midyear, PIA End-of-Year, ETS 12B)
Factors: Size (4), State (2)

Cell Means

Size	State	PIAMid	PIAEnd	ETS 12B	Classes (#)
1	Ala.	23.7	23.4	40.0	7
1	Wis.	23.3	23.2	38.7	7
2	Ala.	18.5	22.3	36.5	11
2	Wis.	24.1	23.6	39.5	4
3	Ala.	20.9	22.8	36.2	13
3	Wis.	22.8	22.7	38.2	11
4	Ala.	23.0	23.6	38.7	12
4	Wis.	22.7	23.6	38.6	15

Correlation Matrix (Between Tests)

	PIAMid	PIAEnd	ETS 12B
PIAMid	1.0		
PIAEnd	.79	1.0	
ETS 12B	.83	.89	1.0

Multivariate F

Hypothesis 1: Between sizes 1, 2, 3, and 4
F-Ratio for multivariate test of equality of mean vectors = 1.86
Degrees Freedom: 9 and 170.5
Marginally Significant, $p < 0.06$

Univariate F's

Test	Between Mean Sqs.	Univariate F	p less than
PIAMid	37.15	2.44	0.07 (Marginal)
PIAEnd	4.15	0.26	0.85
ETS 12B	21.06	0.87	0.46

3 Degrees of Freedom for Hypothesis, 72 for Error

Hypothesis 2: Between states (Alabama and Wisconsin)
F-Ratio for multivariate equality of mean vectors = 1.61
Degrees Freedom: 3 and 70
Not Significant, $p < .20$

Univariate F's

Test	Between Mean Sqs.	Univariate F	p less than
PIAMid	32.03	2.11	0.15
PIAEnd	0.30	0.02	0.89
ETS 12B	12.08	0.50	0.48

1 Degree of Freedom for Hypothesis, 72 for Error

MANOVA RUN #4: Grade 1, Group B (PIA Midyear, PIA End-of-Year, ETS 12B)
Factors: Size (2), State (2), Socioeconomic (3)

Cell Means

Size	State	Socioeconomic	PIAMid	PIAEnd	ETS 12B	Classes (#)
3	Ala.	Low	18.6	22.0	33.2	4
4	Ala.	Low	24.0	20.1	33.3	1
3	Wis.	Low	22.3	22.8	38.8	4
4	Wis.	Low	20.4	18.4	31.9	3
3	Ala.	Middle	21.5	21.9	36.6	5
4	Ala.	Middle	24.8	24.9	41.3	5
3	Wis.	Middle	21.8	21.7	36.3	5
4	Wis.	Middle	24.5	25.8	41.0	5
3	Ala.	High	22.5	24.8	38.7	4
4	Ala.	High	21.3	23.1	37.4	6
3	Wis.	High	26.2	25.2	41.6	2
4	Wis.	High	22.5	24.3	39.8	7

Correlation Matrix (Between Tests)

	PIAMid	PIAEnd	ETS 12B
PIAMid	1.0		
PIAEnd	.81	1.0	
ETS 12B	.86	.89	1.0

Multivariate F

Hypothesis 3: Between Socioeconomic Groups (High, Middle, Low)
F-Ratio for Multivariate Test of Equality of mean vectors = 1.61
Degrees Freedom: 6 and 74
Not Significant, $p < 0.16$

Univariate F's

Test	Between Mean Sqs.	Univariate F	p less than
PIAMid	22.78	1.36	0.27
PIAEnd	32.57	1.79	0.18
ETS 12B	79.18	3.21	0.05

2 Degrees of Freedom for Hypothesis, 39 for Error

MANOVA RUN #5: Grade 3, Group A (Stanford, Form W; PIA Midyear; PIA End-of-Year; Stanford, Form Y)
Factors: Size (4), State (2)

Cell Means

Size	State	Stanford Form W	PIAMid	PIAEnd	Stanford Form Y	Classes (#)
1	Ala.	21.9	21.9	25.4	39.1	5
1	Wis.	16.9	23.0	26.9	37.3	6
2	Ala.	20.4	22.4	25.0	39.2	8
2	Wis.	21.6	27.1	31.0	39.8	8
3	Ala.	21.1	24.8	28.4	40.7	14
3	Wis.	20.4	25.2	30.2	40.0	10
4	Ala.	21.3	25.5	28.8	41.5	14
4	Wis.	18.2	22.9	26.9	35.5	12

Correlation Matrix (Between Tests)

	Stanf. W	PIAMid	PIAEnd	Stanf. Y
Stanf W	1			
PIAMid	.71	1		
PIAEnd	.71	.94	1	1
Stanf Y	.71	.82	.86	

Multivariate F (1 covariate eliminated)

Hypothesis 1: Between sizes 1, 2, 3, and 4
F-Ratio for multivariate test of equality of mean vectors = 0.5708
Degrees Freedom = 9 and 160.8
Not significant, p = 0.82

Univariate F's

Test	Between Mean Sq.	Univariate F	p less than
PIAMid	4.43	0.42	0.74
PIAEnd	9.04	0.64	0.59
Stanford Y	4.24	0.17	.92

3 Degrees of Freedom for Hypothesis, 68 for Error.
The covariate, Stanford Form W Test, was eliminated.

Hypothesis 2: Between States (Alabama and Wisconsin)
F-Ratio for multivariate equality at mean vectors = 9.00
Degrees Freedom = 3 and 66
Highly Significant, p < 0.0001

Univariate F's

Test	Between Mean Sq.	Univariate F	p less than
PIAMid	54.17	5.19	0.03*
PIAEnd	164.12	11.64	0.001**
Stanford Y	0.60	0.02	0.88

1 Degree of Freedom for Hypothesis, 68 for Error.
The covariate, Stanford Form W Test, was eliminated.

Discriminant Analysis for Hypothesis 2

Canonical Variate 1: Variance = 0.41; Percent of Canonical Variation = 100

Discriminant Coefficients

Test	Standardized Coefficients
PIAMid	0.55
PIAEnd	-1.86
Stanf. Y	1.03

Discriminant Plot

Other Hypotheses: There were no significant interactions and results are not included.

MANOVA RUN #6: Grade 3, Group A (Stanford, Form W, PIA Midyear; PIA End-of-Year; Stanford, Form Y)
Factors: Size (2), State (2), Socioeconomic (3)

Cell Means

Size	State	Socio-economic	Stanford Form W	PIAMid	PIAEnd	Stanford Form Y	Classes (#)
3	Ala	Low	17.8	21.0	24.7	36.7	5
4	Ala.	Low	18.2	21.9	23.9	35.7	4
3	Wis.	Low	17.5	24.1	30.4	38.5	2
4	Wis.	Low	5.5	10.0	13.0	15.4	1
3	Ala.	Middle	22.7	24.0	27.7	38.9	4
4	Ala.	Middle	22.5	27.3	31.1	42.8	5
3	Wis.	Middle	20.5	24.7	29.6	39.8	7
4	Wis.	Middle	18.0	22.1	25.5	35.0	6
3	Ala.	High	23.3	29.2	32.7	46.2	5
4	Ala.	High	22.4	26.6	30.3	44.9	5
3	Wis.	High	25.0	30.7	34.4	44.5	1
4	Wis.	High	21.1	26.5	31.4	40.3	5

Correlation Matrix (Between Tests)

	Stanf. W	PIAMid	PIAEnd	Stanf. Y
Stanf. W	1			
PIAMid	0.60	1		
PIAEnd	0.68	0.93	1	
Stanf. Y	0.66	0.78	0.85	1

Multivariate F

Hypothesis 3: Between socioeconomic groups (High, Middle, Low)
F-Ratio for multivariate test of equality of mean vectors = 1.23
Degrees Freedom = 6 and 70
Not significant, $p < 0.30$

Univariate F's

Test	Between Mean Sq.	Univariate F	p less than
PIAMid	29.50	2.83	0.07
PIAEnd	17.50	1.32	0.28
Stanford Y	28.92	0.90	0.42

2 Degrees of Freedom for Hypothesis, 37 for Error.
The Covariate, Stanford Form W Pretest, was eliminated.

MANOVA RUN #7: Grade 3, Group B (ETS Form 23B, PIA Midyear, PIA End-of-Year, ETS Form 23A)
Factors: Size (4), State (2)

Cell Means

Size	State	ETS Form 23B	PIAMid	PIAEnd	ETS Form 23A	# Classes
1	Ala.	30.9	22.1	25.1	41.0	5
1	Wis.	34.6	23.4	28.6	44.2	7
2	Ala.	34.1	24.0	27.1	43.0	15
2	Wis.	35.4	24.7	28.9	44.3	11
3	Ala.	34.7	23.6	26.2	42.4	11
3	Wis.	34.1	24.2	26.9	43.4	12
4	Ala.	35.4	25.0	27.9	44.2	13
4	Wis.	33.6	24.5	27.8	43.0	13

Correlation Matrix

	ETS 23B	PIAMid	PIAEnd	ETS 23A
ETS 23B	1.0			
PIAMid	0.76	1.0		
PIAEnd	0.69	0.84	1.0	
ETS 23A	0.67	0.80	0.91	1.0

Multivariate F (1 covariate eliminated)

Hypothesis 1: Between sizes 1, 2, 3, and 4
F-Ratio for multivariate test of equality of mean vectors = 0.65
Degrees Freedom = 9 and 185.1
Not significant, $p < 0.75$

Univariate F's

Test	Between Mean Sq.	Univariate F	p less than
PIAMid	3.37	0.48	0.69
PIAEnd	9.35	0.78	0.51
ETS 23A	2.67	0.23	0.88

3 Degrees of Freedom for Hypothesis, 78 for Error

Hypothesis 2: Between States (Alabama and Wisconsin)
F-Ratio for multivariate test of equality of mean vectors = 0.80
Degrees Freedom = 3 and 76
Not significant, $p < 0.50$

Univariate F's

Test	Between Mean Sq.	Univariate F	p less than
PIAMid	1.33	0.19	0.66
PIAEnd	21.14	1.77	0.19
ETS 23A	6.85	0.58	0.45

1 Degree of Freedom for Hypothesis, 78 for Error

MANOVA RUN #8: Grade 3, Group B (ETS Form 23B, PIA Midyear, PIA End-of-Year, ETS Form 23A)
Factors: Size (2), State (2), Socioeconomic (3)

Cell Means

Size	State	Socio-economic	ETS 23B	PIAMid	PIAEnd	ETS 23A	ETS 23A*	# Classes
3	Ala.	Low	31.8	20.6	23.0	38.4	22.7	4
4	Ala.	Low	32.7	19.6	23.2	40.0	23.8	2
3	Wis.	Low	34.1	24.6	27.5	43.5	26.6	6
4	Wis.	Low	32.4	23.6	28.0	41.4	25.4	4
3	Ala.	Middle	24.4	15.2	17.6	29.4	20.8	1
4	Ala.	Middle	34.9	25.2	28.5	44.4	26.3	4
3	Wis.	Middle	32.5	21.9	24.9	41.1	23.8	3
4	Wis.	Middle	32.6	23.6	26.6	42.5	26.9	6
3	Ala.	High	38.3	27.0	29.8	47.2	27.7	6
4	Ala.	High	36.5	26.4	29.0	45.3	27.8	7
3	Wis.	High	36.0	25.7	27.8	45.4	28.8	5
4	Wis.	High	37.3	27.7	30.0	46.2	26.6	3

*This column contains the estimated cell means with the covariate eliminated.

Correlation Matrix

	ETS 23B	PIAMid	PIAEnd	ETS 23A
ETS 23B	1.0			
PIAMid	0.72	1.0		
PIAEnd	0.72	0.84	1.0	
ETS 23A	0.71	0.77	0.85	1.0

Multivariate F (1 covariate eliminated)

Hypothesis 3: Between Socioeconomic Groups (High, Middle, Low)
F-Ratio for multivariate test of equality of mean vectors = 3.34
Degrees Freedom = 6 and 68
Significant, $p < 0.006$

Univariate F's

Test	Between Mean Sq.	Univariate F	p less than
PIAMid	7.30	1.43	0.25
PIAEnd	1.28	0.21	0.81
ETS 23A	27.12	7.37	0.002*

2 Degrees of Freedom for Hypothesis, 36 for Error

Discriminant Analysis for Hypothesis 3

Canonical Variate 1 = Variance = 0.67; Percent of Canonical Variation = 99.7
Canonical Variate 2 = Variance = 0.002; Percent of Canonical Variation = 0.3

Discriminant Coefficients
Standardized Coefficients

Test	Variate 1	Variate 2
PIAMid	0.32	0.62
PIAEnd	-0.99	0.72
ETS 23A	1.30	-0.38

Discriminant Plot